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Dominik Morbitzer, Vincent Buskens, Stephanie Rosenkranz and Werner Raub

How Farsightedness Affects Network Formation*

Abstract: We develop a theoretical model of network formation where actors are *limitedly* farsighted. In this way we extend current models with a new set of micro-foundations. Computer simulations are used to predict the stable network structures that are likely to emerge under the new assumptions. The co-author model by Jackson/Wolinsky (1996) is used as an example. The co-author model formulates a tension between stability and efficiency when actors are myopic. Limitedly farsighted actors can overcome this tension but only if the network is small enough. Thus, changing the micro-foundations of the network formation model leads to new implications at the macro-level in the sense that different networks are predicted to be stable than for existing micro-foundations.

1. Introduction

Previous research demonstrates how networks affect social and economic life. People find jobs more easily through weak ties (Granovetter 1973; De Graaf and Flap 1988; Mouw 2003). Artists perform better if they are organized in networks that are neither too dense nor too sparse (Uzzi 2008). Firms are more innovative if they organize their strategic alliances well (Stuart 1998). A so-called broker benefits from being in a position between otherwise not connected actors (Burt 1992).

Because social relationships can be beneficial and people are to some degree aware of the relational structure between them (Krackhardt 1987), actors have incentives to strategically invest in their relations and might try to obtain an optimal position within such a network (Burt 1992; Flap 2002). People nowadays use the term *networking* to describe this strategic behavior. Only more recently, researchers started to investigate, how and why specific network structures emerge assuming that people have the discretion to change their network. Individuals choose with whom they are friends (Van Duijn et al. 2003; Van de Bunt et al. 1999), firms decide with whom they form alliances (Gulati 1995). Strategic models of network formation are developed for analyzing these pro-

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cesses, using game-theoretic tools to predict which type of network structure emerges in these interactions (see Bala/Goyal 2000; Jackson/Wolinsky 1996).

The models that are developed can be distinguished in terms of their assumptions about how actors make their networking decisions given the macro-conditions in which network formation takes place. We refer to these assumptions about how actors make decisions as the 'microfoundations' of the model (see Raub et al. 2011). One group of models considers network formation as a dynamic process in which pairs of actors decide sequentially on whether to change the relation between them (Jackson/Watts 2002). A network is considered stable when no pair of actors wants to change their relation anymore. In most of these dynamic models, it is assumed that actors make their decisions myopically, implying that they neglect subsequent decisions of other actors: actors look one step ahead and only consider whether they are better off after adding or severing one link, playing so called myopic best response. Yet, adding or severing one link might lead to the subsequent addition or severance of other links. Therefore, actors might also use more complex heuristics to determine which links with others they want to change, e.g., actors might use more farsighted strategies in these processes, taking subsequent behavior of others into account. Farsighted actors look more than one step ahead, and anticipate subsequent changes after an initial decision. They then choose a response that produces the best anticipated outcome.

In the literature on network formation there are already some extensions of the myopia model. One alternative is to go to the other extreme and assume that actors are perfectly farsighted (Page et al. 2005; Dutta et al. 2005; Herings et al. 2009; Pantz 2006). This implies that actors consider sequences of changes in the network of arbitrary length towards a given destination network. Such a sequence of changes is feasible if all actors who determine a change in the intermediate steps prefer their position in the destination network over their network position at the moment they implement the change. Here, the assumption that rationality is common knowledge is crucial. The increasing complexity of considering such sequences in larger networks makes the assumption of perfectly farsighted actors unrealistic. Jackson (2003) indicates that perfect farsightedness might be feasible only in very small networks (networks of size 2 to 4). We define networks with $n > 4$ as 'larger'. The number of possible different network structure increases drastically as networks become larger and with that the complexity of the decision situation.[1] We argue that limited farsightedness is a plausible assumption for such settings. We further argue that the assumption of limited farsightedness does not solely depend on network size but can be applied to small and large networks. In addition, the existing models that apply perfect farsightedness do not provide clear cut predictions for arbitrary networks, but mostly provide general theorems on conditions for stability with some examples for specific utility functions in small networks. We know of only one paper by Berninghaus et al. (2012) that tries to model limited farsightedness

[1] See for instance *table 2* which reports the number of non-isomorph network structures for each network size. Note that for $n = 5$ there are already 34 different network structures an actor would have to consider.

in a coordination game with network formation. Therefore, we feel that there is still considerable work to be done to further develop the microfoundations of network formation models particularly in the direction of limited farsightedness.

Experimental research on network formation shows that myopia models sometimes fail to predict empirically observed outcomes (Callander/Plott 2005; Pantz 2006; Berninghaus et al. 2012; Van Dolder/Buskens 2008; Corten/Buskens 2010). The observed deviations from myopic best response behavior may be explained with *anticipatory* behavior of actors. In addition, we know from other experiments on strategic decision-making, specifically on iterated reasoning, that subjects indeed tend to anticipate others' behavior, but in a *limited* way (see Camerer 2003). To investigate iterated reasoning of subjects, when putting themselves into the shoes of others, researchers apply (dominance-solvable) games such as the beauty contest game (see Nagel 1999, for a survey). Camerer (2003, ch. 5) shows that in such games in which it is rather easy to reason ahead, actors still are limited in that respect. In the beauty contest game, actors have to choose numbers between 0 and 100. The actor who chose the number closest to some proportion p of the average number chosen by all actors wins a prize.[2] Stahl and Wilson (1995) and later Camerer et al. (2004) developed a model (the so-called cognitive hierarchy model), which assumes that actors think n steps ahead, implying that actors assume that other actors think only $n-1$ steps ahead. By applying this model for different values of n, they could show with experimental results (mainly on beauty contest games) that, on average, people think two steps ahead.[3] The results demonstrate that models of human action assuming limited farsightedness predict the behavior of humans in some situations better than, for instance, standard Nash equilibrium predictions, i.e., perfect farsightedness (for more references see Nagel 1999; Costa-Gomes et al. 2001; Camerer 2003; Camerer/Fehr 2006).

For developing our model, we build on these assumptions and findings, but change the context of the game to networks; we ask how farsighted people act in strategic *network* situations. There are qualitative differences between decision-making within a network as compared to most game-theoretic situations in which farsightedness has been studied so far. The network formation process is not a straightforward dominance-solvable game where actors anticipate over others' choices and make their (simultaneous) decisions based on what they belief others will choose. A *dynamic* network formation game consists of many rounds of pairs of actors updating their relations sequentially, therefore a farsighted actor has to consider many different actions of many others while the others are typically not in equivalent positions and the order of who changes links is uncertain. So the cognitive demand is even higher in network formation games. Therefore, we develop a simple form of farsightedness, assuming that actors do not look more than two or three steps ahead.

[2] The only Nash equilibrium in the beauty contest is all actors choosing 0. If $p = 2/3$, many people choose numbers around 33. This can be seen as thinking one step ahead and assuming others choose randomly, which implies the others choose 50 on average. The most common choice is numbers around 22, which can be seen as thinking two steps ahead.

[3] This result holds for beauty contest games. In general, the level of reasoning in other dominance-solvable games is between two and three steps.

Summarizing, our research question is: How does the assumption of *limitedly* farsighted actors affect predictions concerning emerging networks and the efficiency of these structures? We compare the model of network formation with myopic actors (looking only one step ahead towards the immediate change in outcomes from changing the network position) with models in which actors look two or three steps ahead, following a path of possible subsequent network decisions. In this way, we can determine how changing the microfoundations affects macro-outcomes.

The model will be outlined in *section 2*. *Section 3* illustrates the model using the co-author model introduced by Jackson and Wolinsky (1996) and shows that different networks are stable under the assumption of limited farsightedness compared to the stable networks when actors are assumed to be myopic. Using computer simulations, we enumerate all stable networks for different sets of assumptions for network size 3 through 8. We use simulation techniques as a theoretical tool since the complexity of our model assumptions makes the derivation of analytical results largely unfeasible. Using simulation techniques provides the advantage that we are able to predict the likelihood of different stable networks to emerge when multiple stable networks exist. The simulation procedure is explained in *section 4*, while the results are summarized in *section 5*. *Section 6* concludes and illustrates possibilities for further research.

2. Model

In the following section we describe our network formation model and how actors make their networking decisions.

2.1 Actors, Networks, Stability, and Efficiency

The set $N = \{1, \ldots, n\}$ is the set of nodes representing actors. A network g indicates which actors in N are connected via a link. Formally, g is a set of unordered pairs of actors $\{i, j\}$. For any pair i and j, $\{i, j\} \in g$ indicates that i and j are linked in the network g; otherwise $\{i, j\} \notin g$. Links are undirected, if i has a link with j then j also is linked with i. We denote the link $\{i, j\}$ also with ij. Let $g + ij$ denote the network obtained by adding the link ij to the existing network g and let $g - ij$ denote the network obtained by deleting the link ij from the existing network. We define g^{ij} as the adjacent network obtained by either adding or deleting a link in g. Thus,

$$g^{ij} = \begin{cases} g + ij, & \text{if } ij \notin g \\ g - ij, & \text{if } ij \in g \end{cases}$$

The utility function vector $u : G(n) \to \mathbb{R}^n$ models the overall benefit net of costs of the actors in a network, where $G(n)$ is the set of all possible networks with n actors. We represent the utility of actor i in network g by $u_i(g)$. The network stability concept we start from is proposed by Jackson and Wolinsky (1996). A network g is *myopically pairwise* stable if

1. $\forall ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$
2. $\forall ij \notin g$, if $u_i(g + ij) > u_i(g)$ then $u_j(g + ij) < u_j(g)$

In words, a network is myopically pairwise stable if no actor wants to severe a link and no pair of actors wants to add a link. The first part of the definition captures the idea that a link can only remain in a network if both actors want to have this link. The second part captures the idea that a link cannot be added to the network if only one of the actors in a dyad wants to add that link. We added the qualification 'myopic' to this definition, because stability is completely based on the direct consequences of a relational change. The addition allows us to distinguish it from farsighted pairwise stability defined below.[4]

To address efficiency, one could consider Pareto efficiency, but the disadvantage is that there often exist many Pareto incomparable states. Alternatively one can consider efficiency based on the sum of utilities of all actors. This aggregate measure is common in the literature (see e.g. Jackson 2008), however, requires interpersonal comparison of utility. Next to this sum of utilities, we assess efficiency by investigating the extent to which actors are able to avoid the inefficient complete network as we show below.

2.2 The Utility Function: The Co-author Model

The method developed in this paper can be applied to any utility function based on a network structure as defined above. In the following, we use the co-author model by Jackson and Wolinsky (1996) to illustrate our formalization of limited farsightedness. In the co-author model, the utility function is based on a setting in which researchers collaborate with each other on research papers. Actors prefer to have many direct links with neighbors who only have few links. The fewer links an actor has, the more time he can spend on each link separately. Actors are thus in competition with others' indirect links (Jackson 2008). Denote the degree of actor i, which is the number of links of actor i, as n_i. Then formally, the utility for actor i in the co-author model is given by

$$u_i(g) = \sum_{j:ij \in g} \left[\frac{1}{n_i} + \frac{1}{n_j} + \frac{1}{n_i n_j} \right].$$

The formalization implies that actors distribute time equally over their links. The last fraction of the equation captures the synergy between the two researchers; if the actors spend more time on each others' project they generate

[4] There are some other limitations to the concept of myopic pairwise stability addressed for instance by Buskens/Van de Rijt 2008 and Jackson 2008. First, the concept only considers deviations on a single link at a time. It is not possible that a player severs several links simultaneously. Second, it considers only deviations by at most a pair of actors. There are models that allow for coalition-wise deviations, and there are models of multiple link deviations at a time that strengthen the weak notion of myopic pairwise stability (Jackson/Van den Nouweland 2005; Buskens/Van de Rijt 2008). Besides several extension of myopic pairwise stability, there are also different stability resp. equilibrium notions such as pairwise Nash stability (for a discussion see e.g. Bloch/Jackson 2006).

more synergy. An actor's utility only depends on the actor's own degree and the degree of his neighbors. The model produces negative externalities, because if actor i creates a link to actor j, this reduces the utility of the actors actor i was already related to as well as the actors actor j was already related to. The co-author model also implies a tension between efficiency and stability, because the gains of the actors who create a new link are mostly less than the losses of the other actors. Still, immediate network gains of creating links imply individual short-term incentives that lead to an inefficient network in the long run. For a broader discussion on this 'tension' between stability and efficiency see Jackson (2008); Buechel and Hellmann (2012). Jackson and Wolinsky (1996) show that in the co-author model the efficient network structure consists of pairs of actors only linked to each other and that myopically pairwise stable networks contain more links and, as a consequence, are inefficient. Buechel and Hellmann (2012) generalize this result, showing that utility models that produce negative externalities in general lead to overconnected networks. In the co-author model, the network formation process with myopic actors almost exclusively leads to the inefficient complete network.

To illustrate that there might be other stable networks under non-myopic behavior, consider the following example. *Figure 1* shows a sequence of three adjacent networks with four actors. The utilities, indicated by the numbers next to the nodes, are taken from the co-author model. Arrows that lead from one network to another show the direction of the formation process when actors make myopic decisions.

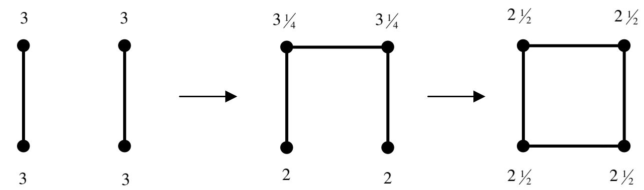


Figure 1: Improving path assuming myopic decision-making

If the (efficient) network on the left is the starting network, adding the link in the network in the middle is a myopic best response for two actors, because their utility raises by 0.25 points. In the subsequent step, the other two actors also form a link between them and they all reach the network on the right where everybody receives a payoff of 2.5. Compared to the starting network, all actors are worse off. This illustrates the tension between efficient and stable networks (assuming myopic best response behavior) as shown by Jackson and Wolinsky (1996). Let us now consider actors who are looking ahead. Farsighted actors anticipate the reactions of others and themselves and are able to foresee how the process might evolve. If the two actors who add the first link in the example above anticipate the reaction of the other two actors, they would prefer to stay in the first network.

2.3 Farsighted Actors and Network Stability

The general idea we introduce is that farsighted actors anticipate the network formation process that arises through altering the network structure by themselves and others. Farsighted actors compare the payoff they get from the current network g to an anticipated network g'. We assume that actors ignore the payoff of intermediate steps since they perceive these states as transient states in which they expect to stay only a negligible amount of time. Intermediate network steps are not considered in the utility function of actors.[5] The set of networks g' that they consider to be relevant depends on the number of steps they look ahead in the network formation process. To formalize this idea we need the concept of a *myopic improving path* (Jackson/Watts 2002).

Following the definition of Jackson and Watts (2002), a myopic improving path from a network g to a network $g' \neq g$ is a finite sequence of networks $g_1, g_2, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that, for every $k \in \{1, 2, \ldots, K-1\}$, either

1. $g_{k+1} = g_k - ij$ for some ij such that $u_i(g_k - ij) > u_i(g_k)$ or $u_j(g_k - ij) > u_j(g_k)$, or
2. $g_{k+1} = g_k + ij$ for some ij such that $u_i(g_k + ij) > u_i(g_k)$ and $u_j(g_k + ij) \geq u_j(g_k)$.

An improving path (see *figure 1*) is a sequence of adjacent networks that can emerge when individuals create or break links based on the improvement the resulting network offers relative to the current one. Each network differs from the previous network by exactly one link. We say that an improving path is a sequence of adjacent networks with the property that g^{ij} 'defeats' its predecessor g for actor i and j, which we denote by $g \prec g^{ij}$.

From the definition of myopic pairwise stability, it immediately follows that a network is myopically pairwise stable if there is no myopic improving path from network g to any other network g'. To establish myopic pairwise stability one needs only to consider one other network g' for comparison with the current network g. In this setting actors only need to know whether adding or severing a link is immediately beneficial.[6]

To extend the concept of myopic pairwise stability to a stability concept based on farsightedness, we need to consider longer sequences of networks. *A farsighted improving path* (Herings et al. 2009; Jackson 2008) from a network g to a network $g' \neq g$ is defined as a finite sequence of networks $g_1, g_2, \ldots, g_K$ with $g_1 = g$ and $g_K = g'$ such that, for every $k \in \{1, 2, \ldots, K-1\}$, either

[5] An example with a discounted stream of payoffs for intermediate steps is found in Dutta et al. 2005.

[6] Jackson and Watts 2002 show that the improving paths do not always converge to a myopic pairwise stable network, but that they might also cycle through a sequence of networks that are repeatedly visited without an option to leave that cycle.

1. $g_{k+1} = g_k - ij$ for some ij such that $u_i(g_K) > u_i(g_k)$ or $u_j(g_K) > u_j(g_k)$, or
2. $g_{k+1} = g_k + ij$ for some ij such that $u_i(g_K) > u_i(g_k)$ and $u_j(g_K) \geq u_j(g_k)$.

Thus, two actors who evaluate an existing or non-existing link between each other change this link if they are better off in the anticipated network g_K at the end of a path of arbitrary length. Jackson (2008) defines the concept of *farsighted pairwise stability*: a network g is farsightedly pairwise stable if there is no farsighted improving path from network g to some network g' such that each pair of consecutive networks along the sequence are adjacent. The idea of a farsighted improving path captures the notion that the actors anticipate all further changes along the path and only compare the final network to the current one, thus neglecting intermediate network utilities (Jackson 2008). This definition of farsightedness assumes perfect farsightedness of actors. Thus, if actors follow an improving path they would not do this unless they anticipate that the endpoint is justified as the resting point of the process. The existence problem is solved by Page et al. (2005); Bhattacharya (2006); Barrientos (2005); Herings et al. (2009) applying the concept of the largest consistent set (LCS) by Chwe (1994) in networks. This solution concept can make predictions for stable networks when actors are farsighted. The LCS defines a set of networks, such that every farsighted improving path that leaves the set has an end point that is again in the set. Therefore, all networks in the LCS can function as resting point.

Meanwhile there exist several different set-based solution concepts. For a good overview of recent papers and a discussion on different approaches of (perfect) farsightedness models see Herings et al. (2009; 2010). Still, all these solution concepts have some common limitations. First, they often do not provide clear cut predictions, because many networks might be within stable sets and the selection problem of multiple predictions arises. Second, the assumption that actors look many steps ahead, certainly in somewhat larger networks, seems less plausible. Third, the farsighted stability concepts mentioned above largely ignore the process of network formation. E.g., although there might be a farsighted improving path from one network to another, depending on the utility function of the network, it might be plausible that along the way some actors have an incentive to deviate from this specific improving path and aim for another network.

We address these three limitations by developing a stability concept assuming *limitedly* farsighted actors. First, we limit farsightedness by defining an improving path that only considers a limited sequence of adjacent networks, Thus, a sequence of networks not of arbitrary length, as in the definition of farsighted improving paths above, but with an appointed length. The myopic improving path is a special case in which only sequences of length 2 are considered and can, therefore, also be called the one-step improving path. We generalize this idea by formalizing the K-step improving path: A *K-step improving path* from network g_1 to a network $g_{K'+1} \neq g_1$ is a sequence of at most $K+1$ networks $g_1, g_2, \ldots, g_{K'+1}$ such that for every $k \in \{1, 2, \ldots, K'+1\}$, either

1. $g_{k+1} = g_k - ij$ for some ij such that $u_i(g_{K'+1}) > u_i(g_k)$ or $u_j(g_{K'+1}) > u_j(g_k)$, or

2. $g_{k+1} = g_k + ij$ for some ij such that $u_i(g_{K'+1}) > u_i(g_k)$ and $u_j(g_{K'+1}) \geq u_j(g_k)$.

Some remarks are needed to clarify this definition. First, it should be clear that if $K = 1$, this is precisely the definition of a myopic improvement path. Second, the definition differs from the farsighted improving path in the sense that the maximal number of changes in the network that is considered in the improving path is K and *not any arbitrary* number. Third, we need to distinguish K' and K in this definition to allow also for shorter improving paths. Imagine that we consider two-step improving paths, but at some occasion only a myopic improvement is possible after which no further myopic improvement is possible. Then, we still expect actors who might look two steps ahead to implement this myopic improvement. Fourth, realize that the last change in a K-step improving path is a simple myopic improvement. Similarly, if $K > 2$, the last two steps in the K-step improving path correspond with steps in a two-step improving paths. In other words, the decision of actors to change the first link in a K-step improving path can be understood as if these actors assume that the subsequent actors will follow at most a $K - 1$-step improving path, and so on. In this way, our formalization directly links to other decision-making models in which actors are modeled as if they believe to think ahead one step more than their partners in the interaction. See, for example, the related assumptions formulated in Stahl and Wilson (1995) on player's models of other players or in the cognitive hierarchy model by Camerer et al. (2004). In these models, it is also assumed that every actor believes he understands the game better than all the other actors. This inconsistency between own and assumed other behavior is consistent with psychological evidence of persistent overconfidence about relative skills in many different domains (e.g. Camerer/Lovallo 1999) and can be proven to be an evolutionary stable strategy (Johnson/Fowler 2011). In line with this, if we assume actors who think two steps ahead ($K = 2$) it implies that every actor believes that the other actors are thinking $K - 1 = 1$ step ahead, therefore playing myopic best response. In that sense, actors who think two steps ahead play *anticipatory* best responses.

We use the concept of a K-step improving path to also define K-step pairwise stability. The idea is that actors look K steps ahead in the network and based on that evaluate whether changing relations might improve their utility. Note that also the K-step improving path implies that links are created if both actors want the link, but that links can be severed unilaterally. The problem is that although actors might see a particular improving path that indeed might improve their position, there might be other improving paths from intermediate networks that deteriorate their position. Why should actors who implement subsequent changes follow the improving path that the actors who made the initial change might have had in mind? After actors who considered a two-step improving path have changed a relation, there are probably many possible myopic improvements for the other actors. Some of these improvements might be on an improving path considered from the situation of the initial actors, but some subsequent improvements, might also be detrimental for the initial actors. (Of course, if we considered the deletion of links, this argumentation only needs to apply to one initial actor.)

Therefore, we need to formalize in addition, how actors weight improving paths that might follow their change, but which are not necessarily improving paths from the actor's perspective who made the initial change. We consider three alternative decision rules to resolve this issue. Note that also with respect to these decision rules we assume homogeneity among the actors to determine stable networks later on. Thus, the stability notions below will be based on the assumption that all actors are to the same extent limitedly farsighted and follow the same decision rules.

Before we can introduce the decision rules, we need to formalize the set of possible paths an actor might consider when anticipating at most K changes in the network. We formalize this in a recursive manner. First we need to define the set of target networks, which is the set of networks an actor i in network g who thinks K steps ahead considers as possible end points after changing a relation with actor j: $M_g^K(ij)$. In addition, we introduce notation for the utility that an actor i assigns to this set of possible target networks, i.e., the utility that i expects from changing the relation with j: $U_i^K(g^{ij})$. For example, if $K = 1$ (myopic actors), then $M_g^1(ij) = \{g^{ij}\}$ and $U_i^1(g^{ij}) = u_i(g^{ij})$. We can already determine the utility that i assigns to this set of networks, because the set contains only one network in case of myopic improvements. We specify this utility further below.

Analogously to the definition of g^{ij} defeating g for myopic improvements we can now also define that g^{ij} defeats g for K-step improvements if in case of adding a link: $U_i^K(g^{ij}) > u_i(g)$ and $U_j^K(g^{ij}) \geq u_j(g)$; and in case of removing a link $U_i^K(g^{ij}) > u_i(g)$ or $U_j^K(g^{ij}) > u_j(g)$. We denote this as $g \prec_K g^{ij}$.

Subsequently, we can define the relevant set of networks to be considered by actors who look two steps ahead:

$$M_g^2(i_1j_1) = \begin{cases} \bigcup_{\substack{i_2,j_2 \neq i_1,j_1 \\ g^{i_1,j_1} \prec_1 g^{i_1j_1,i_2j_2}}} M^1_{g^{i_1j_1}}(i_2j_2), & \text{if there is a } i_2, j_2 \text{ such that} \\ & g^{i_1j_1} \prec_1 g^{i_1j_1,i_2j_2} \\ g^{i_1j_1}, & \text{otherwise.} \end{cases}$$

This is the set of all myopic improvements after the link i_1, j_1 would be changed. If there are no myopic changes anticipated, then only direct myopic improvements are considered. Furthermore, we can define for actors who look three steps ahead:

$$M_g^3(i_1j_1) = \bigcup_{\substack{i_2,j_2 \neq i_1,j_1 \\ g^{i_1j_1} \prec_2 g^{i_1j_1,i_2j_2}}} M^2_{g^{i_1j_1}}(i_2j_2 \leftarrow i_1j_1).$$

Here $M^2_{g^{i_1j_1}}(i_2j_2 \leftarrow i_1j_1)$ is the relevant set of networks to be considered by actors who look two steps ahead from $g^{i_1j_1}$ and consider changing $i_2, j_2 \neq i_1, j_1$.

The notation is slightly different because the myopic improvement that these actors consider beyond $g^{i_1j_1,i_2j_2}$ should not include changing i_1j_1 again.

And more generally we can write:

$$M_g^K(i_1j_1) = \bigcup_{\substack{i_2,j_2 \neq i_1,j_1 \\ g^{i_1j_1} \prec_{K-1} g^{i_1j_1,i_2j_2}}} M_{g^{i_1j_1}}^{K-1}(i_kj_k \leftarrow i_{k-1}j_{k-1} \leftarrow \ldots \leftarrow i_1j_1).$$

The complexity of notation is due to the assumption that actors can consider shorter path lengths while anticipating future outcomes. For example, actors who look K steps ahead might only consider a myopic improvement, assuming that there are no subsequent linking changes. A step-by-step description of this formation process can be found in *section 4* where the simulation model is presented.

Because the decision of the actors in the network involves strategic uncertainty about which subsequent path of changes in the network will be chosen, this decision resembles a strategic decision-making situation that involves risk. Consider a process in which in each round one pair of actors is chosen randomly to evaluate a link. This implies that there are well-defined probabilities about how the network will evolve given what actors assume about how others evaluate links. We present three different decision rules for actors in such situations that are well known for decisions under risk (see Von Neumann/Morgenstern 1944; Luce/Raiffa 1958). These decision rules specify the value actors assign to the set of possible network positions they might reach in K steps.

1. *The maximax decision rule.* Actors focus completely on the best case for this decision rule. Actors' utility in the current networks is compared to the maximum utility possible in all networks that are considered as endpoints. In terms of improving paths, this implies that if a change is on some K-step improving path for the initiating actors, they implement this change. We formalize that by assuming that the utility that an actor i derives from changing a link ij in network g equals his maximal possible value in the networks under consideration:

$$\text{MMAX}_i^K(g^{ij}) = \max_{h \in M_g^K(ij)} u_i(h).$$

2. *The maximin decision rule.* Actors evaluate the situation looking at the worst case. The utility for a specific change is now completely based on the worst situation among all the networks that are relevant to consider:

$$\text{MMIN}_i^K(g^{ij}) = \min_{h \in M_g^K(ij)} u_i(h).$$

3. *The decision rule based on the 'principle of insufficient reason' (PIR).* Here, actors consider all networks that might be reached equally likely and calculate the *mean* of all possible utilities. It is not possible to give a closed formula for the general K-step case. The utility of a change is formalized as

the expected utility based on all possible networks that might be reached in two or three steps, which clarifies how the PIR decision rule works:

$$\mathrm{PIR}_i^2(g^{ij}) = \frac{\sum_{h \in M_g^2(ij)} u_i(h)}{|M_g^2(ij)|}.$$

The calculation is more complex for the three-step procedure:

$$\mathrm{PIR}_i^3(g^{ij}) = \frac{\sum_{\substack{kl \neq ij \\ g^{ij} \prec_2 g^{ij,kl}}} \frac{\sum_{h \in M^2_{g^{ij}}(kl \leftarrow ij)} u_i(h)}{|M^2_{g^{ij}}(kl \leftarrow ij)|}}{|\{kl \neq ij | g^{ij} \prec_2 g^{ij,kl}\}|},$$

where the vertical lines indicate the cardinality of the sets. The formalization implies that at every next level of thinking each possible change is considered equally likely.

These decision rules model individual risk preferences in their most extreme appearances (see Luce/Raiffa 1958). Actors who consider a change are uncertain which pair of actors is picked in the subsequent round(s) and are therefore uncertain which path the formation process follows. We interpret the maximin decision rule as a way of modeling extreme risk aversion, the PIR decision rule as risk neutrality and the maximax decision rule as extremely risk seeking behavior. In the vast literature on decision-making involving strategic risk, not much is known about how individual risk preferences affect decisions in complex strategic situations like network formation. We start with these simplified forms of modeling risk preferences to investigate how sensitive the predictions are to varying risk preferences.

Consequently, three risk-related decision rules for limitedly farsighted behavior are analyzed in how predictions differ. We assume that for each decision rule actors are homogeneous, so there are either only risk-averse, only risk-neutral, or only risk-seeking actors. Note that the sets $M_g^K(ij)$ in the definitions above also depend on the chosen decision rule.

Clearly, the three decision rules that we defined might lead to different sets of stable networks. Still, we can now define straightforwardly K-step stability.

A network g is *K-step pairwise stable* if for a given specification of the utility function U^K from a given set of foreseeable networks $M_g^K(ij)$ there is no pair of actors i and j such that $(g \prec_K g^{ij})$.

Some remarks are appropriate related to this definition of K-step pairwise stability. First, the ordering of networks is not complete. If actors move from network g to g^{ij}, they might want to move back again after the change, because they overlook a different set of networks after the move. Given the utility functions it is clear that the set of MMAX K-step pairwise stable networks is a subset of the PIR K-step stable networks, which is again a subset of a the MMIN K-step stable networks. The reason is that if actors do not anticipate *any* network in which they might be better off by changing a link (MMAX), they certainly do not see any improvements if they apply stricter rules on when they want to change as PIR or, even stricter, MMIN.

3. Farsightedness in the Co-author Model

Figure 2 shows the metanetwork (as introduced by Willer 2007) representing all non-isomorphic network structures with $n = 4$ and utilities given by the co-author model. The arrows in the metanetwork of networks represent possible transitions from one to the other network if actors would be myopic. From this metanetwork, it can be inferred that network K, the complete network, is the myopically pairwise stable network, because that is the only network without outward pointing arrows. We now describe the predictions when actors are looking ahead.

3.1 Looking Two Steps Ahead

To identify the stable networks when actors look two steps ahead, we can also use the metanetwork for myopic improvements. Actors who look two steps ahead place themselves in the shoes of the other actors, assuming these actors are myopic. They follow two arrows in the metanetwork anticipating subsequent changes. For example, the two actors at the top of network B who consider moving from B to C realize that they eventually might end up in network E, F, or G. Now there are two more networks that can be stable. First, the circle network (I) is stable under the maximin, PIR, and maximax decision rule. This can be seen as follows. Actors who look two steps ahead expect that if they would move to network J, the process would continue to network K. This is also the only network to be considered from network J. In network K, the two actors who initiated the change from I to J are both worse off compared to network I. Therefore, actors who look two steps ahead do not want to move from network I to J. The only other change that has to be considered from network I is the path through network G to H. In this case, again, both actors who initiated the change from I to G are worse off in network H. The efficient two-dyads network (D) is stable only under the maximin and PIR decision rules. The most crucial change to be considered is whether two actors would like to connect to network G. Looking forward, they then expect that they will reach network I or H. In network I, they would both earn 2.5, while in network H, one of them will earn 2.25 and one will earn 3.67. This is not attractive under the maximin and the PIR decision rule, because the worst that can happen is that one earns $2.25 < 3$. Also the average of these three outcomes is below 3. However, because the best outcome is $3.67 > 3$, the actors do want to move to network G under the maximax decision rule. The complete network (K) remains stable for all three decision rules, because removing a relation can only bring them back to network K taking myopic subsequent improvements into account.

In a similar way, one could investigate, for each of the three decision rules, from which network actors who look two steps ahead are willing to move. Based on that one could also draw the metanetwork for actors looking two steps ahead. This metanetwork would immediately reveal the stable networks as the networks without outgoing links.

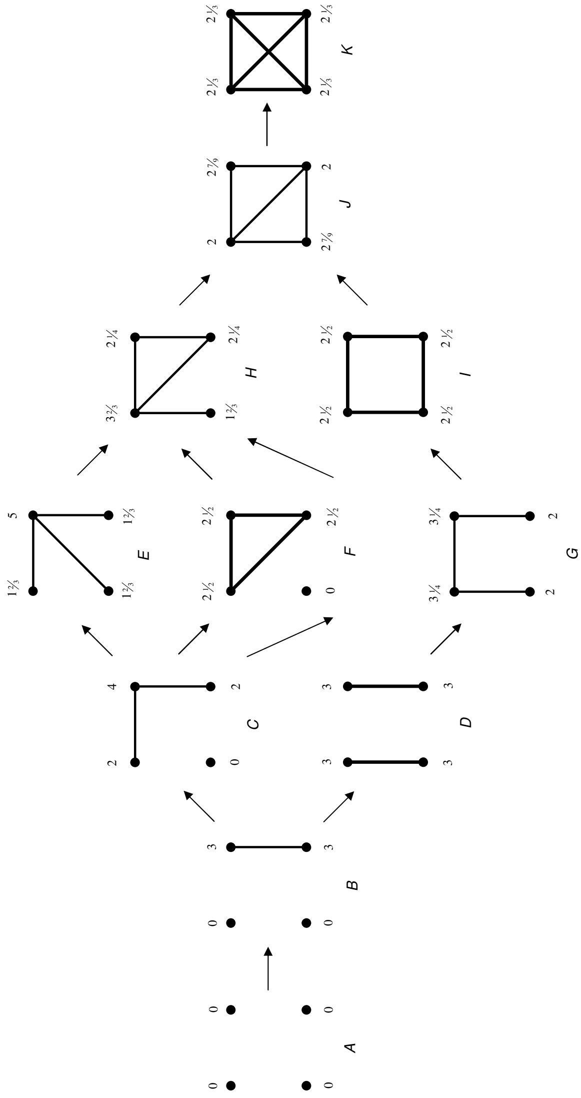


Figure 2: Co-author model with $n = 4$, all non-isomorph networks

3.2 Looking Three Steps Ahead

If actors look three steps ahead, the metanetwork is not a convenient instrument anymore to check for stable networks. One would need two metanetworks, one for myopic actors and one for actors looking two steps ahead. Combining these two metanetworks, the anticipated changes after a change by an actor who looks three steps ahead, can be investigated. This, however, would be a tedious and error-prone job that can better be done by a computer program developed to systematically check all these steps. Therefore, we continue by describing the computer program used to check the stability of networks for the different types of decision rules explained above and to simulate the dynamic process in the network that ultimately converges to a stable network for actors looking two and three steps ahead.

4. Simulation

There are two reasons why we use simulations to continue the stability analyses of the network formation models introduced above. First, we would like to consider networks with more than 4 actors. Second, we would like to estimate the likelihood that a specific stable network is reached in situations with more than one stable network.

In *section 3*, we identified the stable networks for four-actor networks in the co-author model when actors look two steps ahead. Even for this simple case, we neither proved that the networks we claimed to be stable are the only stable ones, nor did we explain for all possible deviations from the stable networks that they indeed were not attractive for actors who look two steps ahead. The simulation provides us with the opportunity to check for every possible network with sizes from 3–8 (these are 13,595 non-isomorphic networks, see also Buskens and Van de Rijt 2008) to identify whether they are stable for the different decision rules specified. In addition, the simulation provides a possibility to follow the network updating process from a non-stable network to a stable network.

The simulation method takes the following steps:

1. Start with some network g.

2. Randomly pick a pair of actors $\{i, j\}$ (every pair with equal probability) to check whether they want to change their link.

3. If i and j do want to change the link ij, change the link and return to step 2 for the new network.

4. If i and j do not want to change the link ij, randomly choose another pair of actors until you find two actors that do want to change their link. Change this link and return with the new network to step 2.

5. If there does not exist any pair of actors anymore who want to change their link, the program ends and the final network is a stable network.

What happens in step 2 of the process described above depends on the stability concept under consideration. It might be that the process we describe above does not converge and that the updating of links continues to cycle through a series of networks. This did not occur in any of our simulation.

When we consider the case of myopic actors, then step 2 is simply checking whether both actors are better off if they consider creating a link and whether one of the two is better off if they consider removing a link. When we consider actors who look two steps ahead, however, step 2 of the simulation process becomes more complicated and consists of the following sub-steps:

2a. In the current network g, if the link ij exists, remove ij; otherwise, create ij to reach network g^{ij}.

2b. For all pairs of actors k and l that are not equal to the pair i and j, consider whether network $g^{ij,kl}$ is a myopic improvement over g^{ij} for actors k and l and, thus, whether myopic actors k and l would like to change their link.

2c. If k and l indeed would like to change, store the payoffs that i and j obtain in network $g^{ij,kl}$.

2d. Take the minimum, the mean, or the maximum of all the payoffs for i stored in step 2c, depending on whether the MMIN, PIR, or MMAX decision rule is considered. Do the same for j. In case $ij \notin g$, if the resulting utility of moving to g^{ij} for both i and j is larger than what they earn in g, add the link ij; in case $ij \in g$, if the resulting utility of moving to g^{ij} for either i or j is larger than what they earn in g, remove the link ij. If there are no k and l who want to change in g^{ij}, i and j change from g to g^{ij} if this is a myopic improvement for them.

The implementation of the version of farsightedness where actors look three steps ahead looks similar, but still requires further explanation because of the payoffs that need to be stored in the process:

2a. In the current network g, if the link ij exists, remove ij; otherwise, create ij to reach network g^{ij}.

2b. For all pairs of actors k and l that are not equal to the pair i and j, consider whether network $g^{ij,kl}$ is a two-step ahead improvement over g^{ij} for actors k and l and, thus, whether actors k and l who look two steps ahead would like to change their link.

2c. If k and l indeed would like to change, store the minimum, mean, or maximum payoff (depending on the chosen decision rule) for i of all the networks that are myopic improvements from network $g^{ij,kl}$, excluding changing ij and kl back. Do the same for j. If there are no myopic improvements possible from network $g^{ij,kl}$, but k and l still want to change to $g^{ij,kl}$, store i's and j's payoffs of network $g^{ij,kl}$. Repeat step 2c for all pairs k and l that are not i and j.

2d. Take the minimum, the mean, or the maximum of all the payoffs for i stored in step 2c, depending on the decision rule to be applied. Do the same for j. In case $ij \notin g$, if the resulting utility of moving to g^{ij} for both i and j is larger than what they earn in g, add the link ij; in case $ij \in g$, if the resulting utility of moving to g^{ij} for either i or j is larger than what they earn in g, remove the link ij. If there are no k and l, who want to change in g^{ij}, i and j change from g to g^{ij} if this is a myopic improvement for them.

Because we start in every possible network, we are sure that we determine also every possible stable network for each condition. When we start in a stable network the process immediately stops and because we also save the number of iterations until convergence, it is straightforward to check which network structures are stable.

Because the order of pairs of actors that can evaluate their link is randomly determined, it does not need to be the case that if we start in a given network the process always converges to the same stable network. Therefore, we start the simulation five times from each starting network to determine the likelihood that a specific stable network is reached, based on that we start an equal number of times in every possible non-isomorphic starting network.

5. Results

In this section, we present the results of our simulations. We investigate *which* networks are stable when actors are limitedly farsighted, looking two and three steps ahead, and compare these to predictions for myopic actors. Thereafter, we consider the question how likely it is that certain networks emerge.

5.1 Stable Networks

Table 1 summarizes the number of stable networks under the different stability concepts for each network size. The maximin decision rule produces very high numbers of stable networks. The numbers increase drastically with network size. The maximax decision rule shows the other extreme and has only very few stable networks. This pattern is even clearer when actors look three steps ahead. The reason is that if actors look further ahead, for the maximin decision rule it becomes more and more likely that an actor finds an alternative network in which he is worse off and, therefore, does not want to change. For the maximax decision rule, the opposite happens. Actors almost always find an alternative network in which they would be better off if they look enough steps ahead and, therefore, they hardly ever stop changing the network. Because these two decision rules focus purely on the worst- or best-case scenario, a decision can be based on very unlikely events in a large set of alternative outcomes. Therefore, we argue that the PIR decision rule offers the best interpretable results, as it does not produce such extreme results. Realize that from the simulation, it is clear that, under the maximin decision rule, there should be at least as many stable network as under the PIR decision rule, and that, under the PIR decision rule, there should be at least as many stable networks as under the maximax decision rule. The reason is that the conditions under which actors want to change are most restrictive for the maximin decision rule, less restrictive for the PIR decision rule, and least restrictive for the maximax decision rule.

	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$
myopic (one-step)	1	1	1	2	2	2
two-step						
maximin	2	3	7	14	45	153
PIR	2	3	2	2	3	4
maximax	2	2	1	1	2	2
three-step						
maximin	2	4	7	29	137	981
PIR	2	4	6	6	5	7
maximax	2	2	1	1	1	1

Table 1: Number of stable networks

The complete network is stable for $n = 3$ for all levels of farsightedness and all decision rules. Additionally, we observe a single dyad with one isolated actor as a stable network for actors looking two and three steps ahead, for each decision rule. In this case, the two connected actors (each obtaining a utility of 3) are excluding the single actor such that they will not agree to form a link with him because of the threat of ending worse off in the completely connected network (which brings them a utility of only 2.5).

The stable networks for $n = 4$ were already reported in *section 3*. The efficient network of dyads is stable for the maximin and PIR decision rule, but not for the maximax decision rule. We also find the circle network and the complete network stable under all of the three decision rules and levels of farsightedness.

Remember that for myopic actors the complete network is the only stable network for $n \leq 5$. For $n \geq 6$, there is one additional stable network for myopic actors that consists of a single dyad and completely connected component (from here on labeled as 'dyad+com'). These network structures are also among the stable network predictions when actors look two and three steps ahead. For the maximax decision rule and when actors look three steps ahead, the complete network is the only prediction for $n > 4$. For larger networks, under the maximin and PIR decision rule, the complete network also remains a stable network, but there is a number of other stable networks as well. The more frequent network structures that are stable in the farsightedness models for $n = 5$ are shown in *figure 3*. We discuss these networks in more detail in the following sections where we analyze the likelihood of the emergence of specific networks.

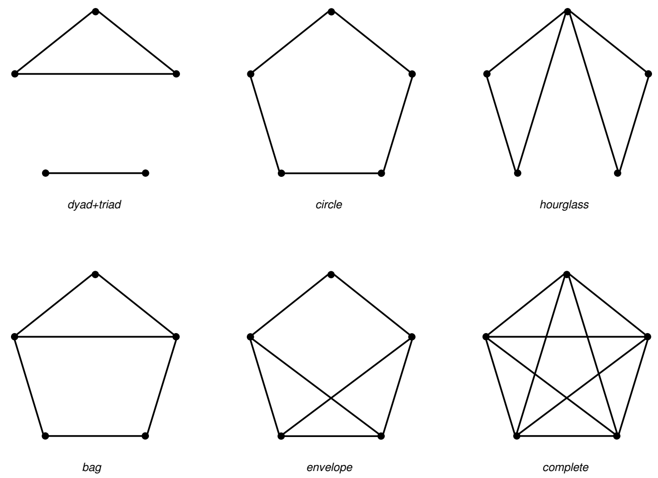


Figure 3: Stable networks for $n = 5$

5.2 The Likelihood of the Emergence of Networks

5.2.1 Looking Two Steps Ahead

We now analyze the likelihood of specific networks to emerge. *Table 2* summarizes the results of the dynamic procedure for all networks from size 3 to 8. The numbers in the rows report the percentage of the simulations runs that converged to a certain network structure. Starting five times from all non-isomorph networks.[7] We display these percentages for the *complete* network, because it is the most common structure we observe, and also for the so-called *dyads* networks, because these are the most efficient structures. Efficient networks consist of either only dyads for even-sized networks or dyads plus a single triad for uneven-sized networks. The row *'other'* (as shown in *table 2*) reports important networks that also emerged frequently (e.g. the label (dyad+com) means that this network consists of one single dyad plus one completely connected component). We further report the average number of *iteration* steps (i.e. link changes) it takes until convergence.

In the upper block of the table the results of the myopia predictions are listed (*myopic*), followed by the three different decision rules for actors who are looking two steps ahead, referred to as *maximin, PIR, maximax.*

[7] We realize that the precise likelihoods depend on the fact that we use every non-isomorphic network only once, while it might make sense to use a weight for the results of each network relative to the number of isomorphisms that exist for each non-isomorphic network. We decided not to introduce this complication, because the main conclusion are not driven by the exact percentages that each stable network emerges.

	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$
myopic (one-step)						
complete	1.0	1.0	1.0	.99	.99	.99
dyads	–	–	–	–	–	–
other	–	–	–	(dyad+com) .01	(dyad+com) .01	(dyad+com) .01
no. of iterations (std. dev.)	1.5(1.1)	3(1.7)	5(2.1)	7.4(2.7)	10(2.9)	14(3.1)
maximin two-step						
complete	.50	.49	.55	.64	.75	.83
dyads	.50	.27	.14	.03	.00	.00
other	–	(circle) .24	(circle) .12	(dyad+com) .08	(dyad+com) .05	(dyad+com) .03
no. of iterations (std. dev.)	.5(.5)	1.4(1.1)	2.5(2.0)	4.7(2.9)	7.8(3.9)	12(4.5)
PIR two-step						
complete	.50	.51	.89	.93	.98	.98
dyads	.50	.11	.11	.00	–	–
other	–	(circle) .38	–	(dyad+com) .07	(dyad+com) .02	(dyad+com) .02
no. of iterations (std. dev.)	.5(.5)	1.8(1.6)	4.3(2.2)	7.0(2.8)	11(5.2)	14(3.4)
maximax two-step						
complete	.50	.25	1.0	1.0	.99	.99
dyads	.50	–	–	–	–	–
other	–	(circle) .75	–	–	(dyad+com) .01	(dyad+com) .01
no. of iterations (std. dev.)	.5(.5)	2.4(2.2)	9.8(6.9)	36(28)	114(97)	468(438)
no. starting networks	4	11	34	156	1,044	12,346

[a]See *figure 3* and the text for an explanation of 'other' networks.

Table 2: Proportions emergence of stable networks: myopic and looking two steps ahead

For actors looking two steps ahead, we make the following general observations: under all decision rules the complete network is the most frequently emerging network. However, in smaller networks, for limitedly farsighted actors, efficient networks emerge in approximately half of the cases. And the larger the network size, the more likely it is that the complete network emerges.

We further describe the detailed results for all decision rules. Under the *maximin* decision rule, stable networks emerge that are not the inefficient complete network, for smaller networks ($n = 3$ and $n = 4$) and in half of the cases. For $n = 4$, efficient dyads and the circle network emerge in 27% and 24% of cases. For $n = 5$, the process runs towards the complete network in 55% of all cases, dyadic networks emerge only in 14% (the efficient dyad+triad network in 6% and the dyads+iso network in 8% of cases), the circle network in 11% of cases. For larger networks ($n \geq 6$), the likelihood that efficient networks evolve, becomes very small (3%, 1% and almost 0% for sizes from 6 to 8) and the complete network appears more often.

Under the *PIR* decision rule, the complete network emerges for $n = 4$ in more than half (55%) of the cases. For a circle and a two-dyad network, the likelihood is 33% and 12%. For $n > 4$, the complete network appears in most cases (in 89% for $n = 5$ to more than 98% of cases for networks $n = 7$ and $n = 8$). For $n \geq 7$, the 'other' network has the same structure as in the myopia model, namely a network with a single dyad and one completely interconnected component. However, this network was never reached from any other initial network.

Under the *maximax* decision rule, the circle network is more likely to appear than the complete network (75%) for $n = 4$. However the complete network emerges in 100% of cases for $n = 5$ and $n = 6$. For $n > 6$, the dyad+com network emerges in only 1% of all simulation cases.

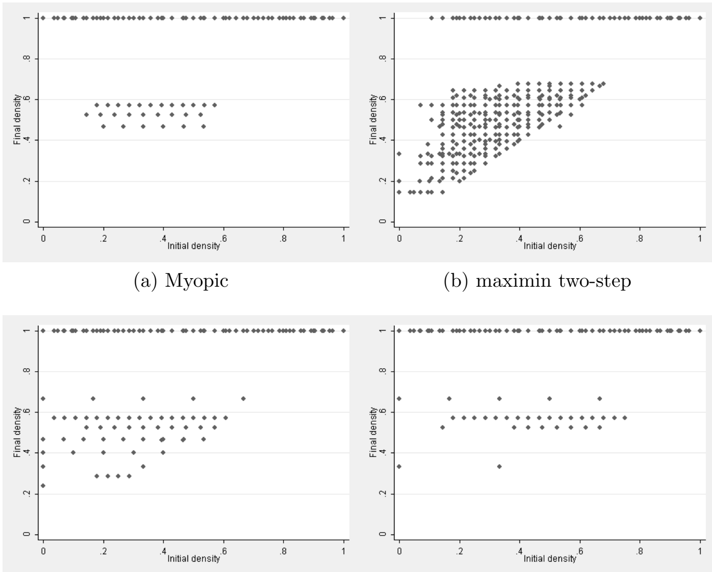
When actors look two steps ahead, there is no state in which the process is cycling. Under the risk seeking decision rule (maximax) to reach a stable network state, the number of link changes is the highest. Actors constantly change the network to achieve a network position that they assume is more beneficial (note the average number of iteration steps).

The scatter plots in *figure 4* reports the formation processes where density[8] of the initial network (on the x-axis) is plotted against the density of the converged network (on the y-axis).[9] All repetitions over all network sizes are displayed. Most formation processes stabilize in the inefficient complete network (density equal to 1). Therefore most points are located on the upper part of the graph. In *figure 4a*, the points lying on the three horizontal lines in the middle are the dyad+com networks that are stable for networks of size 6 to 8. In *figure 4b*, networks that are indicated by points lying on the diagonal, represent cases where the initial network is also the final network. Most stable networks lie above this diagonal, indicating that the networks become more dense until convergence.

[8] The proportion of links in a network relative to the maximal number possible links.

[9] In the co-author model network density is related to efficiency as described above. However the density measure better shows how adding links is the driving force of formation processes in the co-author model. The corresponding efficiency tables are shown in Appendix A.

Note (e.g. in *figure 4b*) that if the initial network is already quite dense (around .7) then the formation processes always stabilize in the inefficient complete network. Formation processes in which the final network is less dense than the initial network are infrequent, since in most situations there are only few incentives for actors to delete links, even when actors look ahead (these cases are the points lying below the diagonal). This appears more often under the maximax decision rule, where actors foresee more networks in which they are better off and therefore tend to delete more links.



(a) Myopic (b) maximin two-step

(c) PIR two-step (d) maximax two-step

Figure 4: Initial network density versus final network density, myopic and two-step

5.2.2 Looking Three Steps Ahead

The detailed results for actors who look three steps ahead are shown in *table 3*. In general we observe a similar pattern as for actors who look two steps ahead, but now the pattern becomes sharper: Still with increasing network size, converged networks tend to become overconnected and, therefore, inefficient. Although the effect of network size is less severe than compared to when actors look two steps ahead. For example, under the PIR decision rule, actors who look three steps ahead will not end up in completely connected networks as often as when looking two steps ahead (for $n = 5$ and $n = 6$ in only around 30% of cases the complete network is stable). In more detail, for $n = 3$ the results remain unchanged as compared to actors who look two steps ahead. Under the maximin decision rule the complete network is often not the final network of the process (it is highest for $n = 4$ with 31% of cases). There are many stable networks when actors use the minimax decision rule. Now looking three steps ahead actors foresee even more network positions where they might end up worse off. Therefore, most processes stop after very few link changes (the number of iteration steps with actors who look three steps ahead is lower than with actors who look two steps ahead). For $n = 4$, one additional network emerges, the network with a triad and an isolated actor (triad+iso) (18% of cases). This is possible because all actors within the triad fear that they might end up worse off after connecting to the isolated actor (looking three steps ahead actors foresee the formation process untill the complete network). For $n = 5$ in 27% the complete network emerges, in 9.4% of cases the efficient dyad+triad network and the two dyads with an isolated actor in 8.8% of cases. Other networks that emerge frequently are the bag network with 23%, and the hour glass in 17% (see *figure 3*). For $n = 6$, the complete network emerges in 16.3% of cases, but in only 2.6% of formation processes actors remain in the efficient dyad structure. Most cases stabilize in the dyad+com network. For $n = 7$, the network that emerges most often has one actor with five connections, four actors with four, one with three and one with two connections ($5^1 4^4 3^1 2^1$) (see Appendix A). For $n = 7$ and $n = 8$, efficient dyads are only stable when they are the starting network configuration.

Under the PIR decision rule, the triad+iso network emerges in 29% of cases for $n = 4$. The circle and efficient network emerge in 27% of all cases. The complete network emerges in 16% of cases. For $n = 5$, the complete network emerges in 26.5% of cases, the dyad+triad network in 8.8%, and the dyads+iso network in 6.5% of cases. Other converged networks for $n = 5$ are the 'envelope' (see *figure 3*) in 22.9% of cases and the circle network (5.9% of cases). For $n = 6$, the complete network appears in 32.4% of cases, the network that emerges most often is an almost completely connected network where three links are missing (the octahedron, see Appendix A). Efficient dyads only emerge when the process also starts in this network. For $n = 7$ and $n = 8$ the complete network emerges in most cases. Efficient networks are unlikely and emerge in less than 1% of cases.

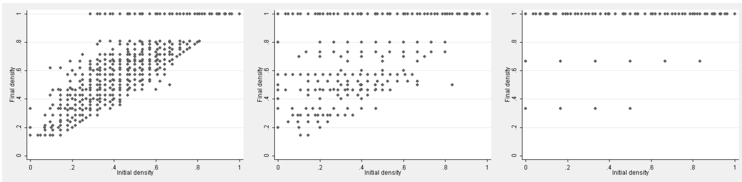
Under the maximax decision rule, there are three converged networks for $n = 4$. The circle network emerges in 60%, the complete network in 15% and the dyad in 25% of cases. For $n > 4$, there is only one stable network, namely the complete network. This can be explained because from every other network actors who use the maximax decision rule anticipate a network position in which they are better off than in their current position. Note the high number of link changes in this procedure.

	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$
maximin three-step						
complete	.50	.31	.27	.16	.09	.22
dyads	.50	.27	.18	.02	.01	.00
other	–	(triad+iso) .18	(bag) .23	(dyad+com) .26	($5^1 4^4 3^1 2^1$) .10	(dyad+com) .01
no. of iterations (std. dev.)	.5(.5)	1(0.9)	2.0(2.5)	5.3(8.5)	3.7(4.7)	5.7(10)
PIR three-step						
complete	.50	.16	.27	.32	.97	.97
dyads	.50	.27	.01	.00	.01	.00
other	–	(triad+iso) .29	(bag) .29	(octah.) .37	(dyad+com) .02	(dyad+com) .02
no. of iterations (std. dev.)	.5(.5)	1.8(2.9)	11(28)	37(71)	206(209)	461(456)
maximax three-step						
complete	.50	.15	1.0	1.0	1.0	1.0
dyads	.50	.25	–	–	–	–
other	–	(circle) .60	–	–	–	–
no. of iterations (std. dev.)	.5(.5)	8.2(8.9)	n.a.	n.a.	n.a.	n.a.
no. starting networks	4	11	34	156	1,044	12,346

[a]See *figure 3*, Appendix A, and the text for an explanation of 'other' networks.

Table 3: Emergence of stable networks: looking three steps ahead

Additionally, we observe cycles in the maximin decision rule for $n = 8$. In 11.2% of the simulation runs the process did not converge to some network. By looking at the starting networks that always end up in non-convergence, we identify the networks that are part of cycles. In our simulations there were 547 different cases of starting networks that always lead to a cycle. Of the networks that are part of cycles, 96% can be described by six different networks structures. These, however, do not have any appealing structural properties we could detect.



(a) maximin three-step (b) PIR three-step (c) maximax three-step

Figure 5: Initial network density versus final network density, three-step

For actors who look three steps ahead, the scatter plots between initial network density and final network density show similar patterns as compared to the plots for actors who look two steps ahead. There are more formation processes where the initial network density is higher than the density of the stable network (points below the diagonal). Also note that under the maximin and PIR decision rule the initial network density can be higher (around .8) before all networks stabilize in the complete network (compared to the version of actors looking two steps ahead where density is around .7. Note the gap in *figures 4b and 5a*). For the PIR decision rule, looking three steps ahead implies more networks that stabilize in intermediate density levels than compared to the two-steps version.

6. Conclusion and Discussion

Theoretical arguments and empirical results have questioned the commonly used assumption of myopic behavior in network formation models. We develop a model in which actors make their decisions in a limitedly farsighted manner, to analyze how this affects the emergence and efficiency of networks. In the model, actors anticipate subsequent decisions of other actors when making their own decisions. Changing the microfoundations of the network formation model implies new predictions at the macro-level in the sense that different networks are predicted to be stable than for myopic behavior.

The main finding is that in *smaller* networks, where limitedly farsighted actors are able to oversee the formation process, it is possible to overcome the tension between efficiency and stability, while this is impossible for myopic actors. However, limitedly farsighted actors cannot fully overcome this tension in larger networks; as the network size increases they still end up in suboptimal situations. Increasing the level of farsightedness to three steps helps to overcome the tension between efficiency and stability in some more cases but also here, as network size increases, actors end up in inefficient structures, predominantly the complete network.

Experimental results for the co-author model show that the emergence of efficient networks, as predicted by the limited farsightedness model, is possible when subjects play this game in the laboratory (see Van Dolder/Buskens 2008). The experimental evidence is hard to reconcile with myopic behavior and may be consistent with limited farsightedness assumptions. On the one hand, some phenomena, e.g., deviations from game-theoretic predictions we observe in experiments or real life situations, can often be better explained assuming some form of limited rationality. On the other hand, experimental researchers claim that their results can be explained by assuming subjects use more sophisticated strategies, and that myopic best response behavior might be a too simple form of modeling bounded rationality (Callander/Plott 2005; Van Dolder/Buskens 2008; Corten/Buskens 2010). With the means of simulation techniques, we were able to predict possible network outcomes when actors are limitedly farsighted for networks from size 3 to 8.

Additional assumptions about how actors evaluate uncertain network outcomes were needed when assuming limited farsightedness in a dynamic formation process. Risk preferences can have a big impact on the formation process and on predictions of stable networks. Risk-averse actors prefer other network changes than risk-neutral or risk-seeking actors. The formation process itself differs in terms of convergence time, i.e., the number of link changes needed to reach a stable state. Risk-seeking actors take far more link changes to get to a stable state. Also they are more likely to move to intermediary and temporarily less beneficial network positions to strive for a perceived better outcome. This might explain some seemingly irrational behavior often observed in experiments. In models of strategic network formation the influence of risk preferences so far seems neglected.[10] Looking at the different predictions, risk preferences seem to be a rather important factor. In a dynamic approach, where actors are indifferent on where the process might evolve, it is inevitable to discuss actors' evaluation of future outcomes in terms of risk preference and how this might influence actors' decisions. Because we only considered very extreme forms of risk-averse and risk-seeking preferences, we could show that risk preferences matter, but we neglected weaker forms of risk averse and risk seeking preferences that might be more realistic in analyzing network formation behavior in empirical applications. More research will be needed to shed light on that topic.

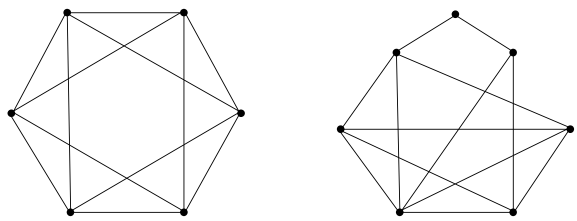
Future research might look as follows: first, other ways in which actors derive utility from networks should be used to study how limited farsightedness affects network formation in different settings (e.g. the connections model, see see Jackson and Wolinsky 1996). Second, experiments on network formation should be conducted to test the model. Third, risk preferences and also heterogeneity of actors should be included into theoretical models to study their effects in more detail. Individual level differences (heterogeneity) is an impor-

[10] We are only aware of one working paper by Kovářík and Van der Leij 2010.

tant factor within network formation.[11] Heterogeneity in our approach can be understood in two ways: first, heterogeneity regarding farsightedness (some actors might look further ahead than others), and second, heterogeneity in terms of risk preferences. It is of interest to investigate both cases of heterogeneity and how they might influence predictions of stable networks. The simulations in this paper only covered networks of size 3 to 8. Applying the model to larger networks would be another extension where the impact of limitedly farsighted actors on network formation can be investigated. Possibly, actors interact limitedly farsighted within their local network but interact myopically with more distant actors in the network.

Appendix A: Network Efficiency

Figure 6 shows infrequently emerging networks for when actors look three steps ahead.

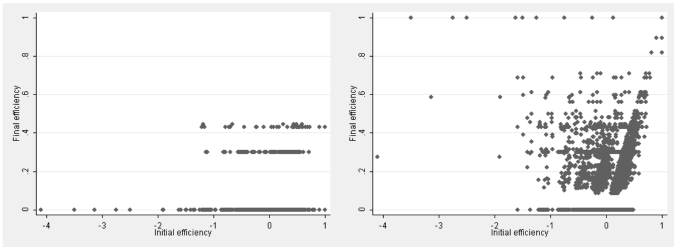


n=6, octahedrone *n=7, $(5^1 4^4 3^1 2^1)$*

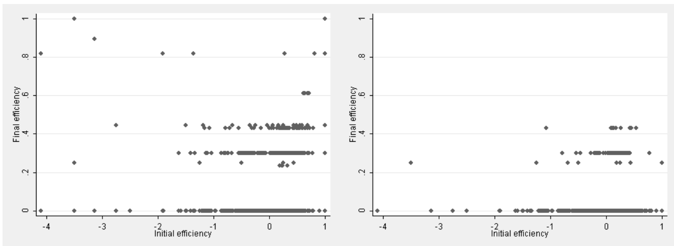
Figure 6: Stable networks for actors looking three steps ahead, $n = 6$ and $n = 7$

[11] Fowler et al. 2009 could explain a large amount of variance within network characteristics such as in-degree through individual differences in genes.

Figures 7 and 8 show the relationship between the efficiency of the initial networks and the final network of the formation process.

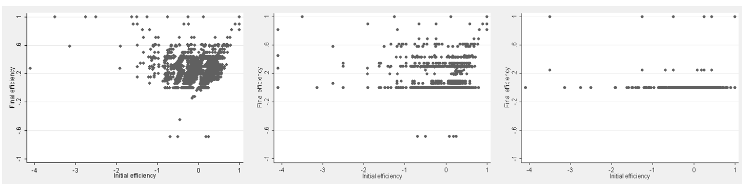


(a) Myopic (b) maximin two-step



(c) PIR two-step (d) maximax two-step

Figure 7: Initial network efficiency versus final network efficiency for $n = 3$ through 8, co-author model with actors who look two steps ahead



(a) maximin three-step (b) PIR three-step (c) maximax three-step

Figure 8: Initial network efficiency versus final network efficiency for $n = 3$ through 8, co-author model with actors who look three steps ahead

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