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A Note on the 'Rationality of Becoming' and Revealed Preference*

Abstract: This note comments on Professor Majumdar's concept of an individual who seeks to change his own preferences over time. It is argued that while one can formulate "revealed preference axioms" which will rationalize the choices of Professor Majumdar's individual, it is unlikely that the choice of such an individual will reveal his preferences.

Majumdar (1980) has provided an interesting departure from the restrictive concept of the economic man whose sole aim in life is to maximize in terms of an exogenously given and unambiguously defined preference ordering, and whose preferences can be inferred by observing his choice. In Majumdar's formulation, the individual does not take his own preference ordering as fixed but tries to mould it over time according to his notion of certain preference orderings (or alternatively, certain types of personalities) being better than other preference orderings (or alternatively, other types of personalities). At the end of his paper Majumdar raises the question of formulating revealed preference axioms to characterize the behaviour of such an individual. The purpose of this comment is to argue that while it may not be very difficult to formulate alternative revealed preference axioms which may characterize the behaviour of such an individual, it remains debatable whether the choice of this individual will *reveal* his preferences in a plausible sense. This, of course, does not in any way contradict Majumdar's analysis; it only makes explicit certain implications of his interesting formulation.

I.

Before proceeding to formulate revealed preference axioms which may characterize Majumdar's rational individual, it may be useful to clarify a methodological point. Sen (1977) and Majumdar mention the possibility of cross examining the individual (in addition to observing his choices in the usual sense of the term) as a method of gaining information about his preferences. However, even when one adopts this method, methodologically one can still distinguish between cross examinations conducted entirely in terms of the language of choice and cross examinations conducted entirely in terms of the language of preference (of course, there could be a hybrid of the two). Thus one might try to gain information about a person's preferences by cross examining him about what his choices would be in alternative

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hypothetical situations, alternatively, one could ask him questions directly about his preferences. In what follows, I assume that information about the individual's preferences is to be culled from data couched entirely in the language of choice, irrespective of whether such data are collected from direct observation of the individual's choices or from cross examination of the individual.

Let there be T time periods $(1, \dots, T)$ before the individual, and let X be the set of all conceivable alternatives which can be presented before the individual in any given time period. A non-empty subset of X will be called an environment. Let C^t be the choice function of the individual in time period t , defined as follows: for every environment A , C^t specifies exactly one non-empty subset $C^t(A)$ of A . $C^t(A)$ has the obvious interpretation of being the choice set or the set of chosen elements from A . For all alternatives x and y , x is *revealed weakly preferred* to y in time period t (alternatively, $x \succsim_t y$) if and only if in time period t , x is chosen from some environment A which contains y ; x is *revealed preferred* to y in time period t (alternatively, $x \succ_t y$) if and only if in time period t , y is rejected while x is chosen from some environment A [i.e., $x \in C^t(A)$ and $y \in (A - C^t(A))$]. Consider the following assumptions:

A.1: For all alternatives x and y , if x is revealed weakly preferred to y in any time period, then y cannot be revealed preferred to x in any time period.

A.2: For all alternatives x and y , if x is revealed weakly preferred to y in any time period, then in the same time period y cannot be revealed preferred to x ; and once the choice function changes in any way between two time periods, it cannot revert back to its original form in any subsequent time period. [More formally, for all $g \in \{1, \dots, T\}$, and for all $x, y \in X$, if $x \succsim_g y$, then $\sim y \succ_g x$; and for all $g, h, k \in \{1, \dots, T\}$ and for every environment A , if $g < h < k$ and if $C^g(A) = C^k(A)$, then $C^g(A) = C^h(A)$.]

A.3: For all alternatives x and y , if x is revealed weakly preferred to y in any time period, then in the same time period y cannot be revealed preferred to x ; and if the choice set from any given environment changes between two time periods, then it cannot revert back to its original form in any subsequent time period. [More formally, for all $g \in \{1, \dots, T\}$ and for all $x, y \in X$, if $x \succsim_g y$, then $\sim y \succ_g x$; and for all $g, h, k \in \{1, \dots, T\}$ and for every environment A , if $g < h < k$, and if $C^g = C^k$, then $C^g = C^h$.]

Since each of these assumptions – A.1, A.2, and A.3, implies Arrow's (1959) Weak Axiom of Revealed Preference with respect to choices in each time period, by the theorem of Arrow (1959) it follows that if any of these three assumptions is satisfied, then for every time period g , \succsim_g must be an ordering and C^g must be "rationalizable" by \succsim_g [i.e., for every environment A , we must have $C^g(A) = \{x \in A \mid x \succsim_g y \text{ for all } y \in A\}$]. It is also clear that A.1 implies A.2 (though the converse is not true) and A.2 implies A.3 (again, the converse is not true).

If A.1 is satisfied it is easy to see that for all time periods g and h , C^g and C^h will be identical (so that the implied orderings \succsim_g and \succsim_h will also be identical).

Clearly, this is not the appropriate assumption in a context where we want to discuss individuals who deliberately seek to change their own personalities and whose choice functions will therefore shift over time. Which of the remaining two assumptions – A.2 and A.3, can be reasonably postulated for Majumdar's individual? If the individual satisfies Assumption 3, then once the implied ordering changes from \geq_g to $\geq_h \neq \geq_g (h > g)$, it cannot again change to $\geq_k = \geq_g (k > h)$. Assumption 2 is much stronger. It implies that for all alternatives x and y and for all time periods g, h and k ($k > h > g$), if \geq_g and \geq_h do not coincide over $\{x, y\}$ then \geq_g and \geq_k cannot coincide over $\{x, y\}$.

Assumption 3 is intuitively reasonable. If $C^1 \neq C^2 \neq C^3$, then it is difficult to see why the individual should have changed from C^1 to C^2 in the first place. If the individual has deliberately changed his choice function, it seems only a wasteful experiment to revert back to the original choice function. On the other hand, the intuitive basis of A.2 is not at all clear. Suppose a world of two commodities – bread and whisky, $X = \{ (4,0), (3,1), (2,2), (1,3), (0,4) \}$ where $(0,4)$ stands for a bundle of commodities containing 4 units of bread and 0 units of whisky and so on. Consider an alcoholic, whose present preference ordering is R (with associated strict preference relation P) where $(0,4)P(1,3)P(2,2)P(3,1)P(4,0)$. Suppose this individual first wants to get rid of his craving for whisky and to settle ultimately for preference ordering R^* (with associated strict preference relation P^*) where $(2,2)P^*(3,1)P^*(4,0)P^*(1,3)P^*(0,4)$. Suppose he finds that to start with he will be absolutely miserable unless he gets at least 3 units of whisky but once he is accustomed to 3 units of whisky, in the next period he can manage (though with still a craving for whisky) with 2 units of whisky and so on. After he continues for some time without consuming any whisky, he would completely get rid of his addiction and then he can consume 'normal' quantities of whisky without reverting back to his original preferences. Given this it may be perfectly rational for him to plan for the sequence of choice functions $(C^1, C^2, C^3, C^4, C^5, C^6)$ where \geq_g implied by $C^g (g=1..6)$ are as follows:

\geq_1	\geq_2	\geq_3	\geq_4	\geq_5	\geq_6
(1,3)	(2,2)	(3,1)	(4,0)	(4,0)	(2,2)
(0,4)	(1,3)	(2,2)	(3,1)	(3,1)	(3,1)
(2,2)	(0,4)	(1,3)	(2,2)	(2,2)	(4,0)
(3,1)	(3,1)	(0,4)	(1,3)	(1,3)	(1,3)
(4,0)	(4,0)	(4,0)	(0,4)	(0,4)	(0,4)

Note that this intuitively plausible sequence of choice functions satisfies A.3. However, it does not satisfy A.2 since $C^3 (\{(2,2), (4,0)\}) = \{(2,2)\} = C^6 (\{(2,2), (4,0)\}) \neq C^4 (\{(2,2), (4,0)\}) = \{(4,0)\}$. In general it would seem that A.3 rather than A.2, is more suited to capture the type of rationality with which Majumdar is concerned.

II.

The example at the end of the last section illustrates another aspect of the problem. Note that though for every $g(1 \leq g \leq 6)$, C^g is rationalizable by an ordering \succsim_g , for every g , \succsim_g need not necessarily coincide with the preference ordering of the individual in time period g . For example when our alcoholic starts out with the choice function C^1 , his preference ordering is R which is different from the ordering \succsim_1 implied by C^1 . A little reflection will show that this is not unusual. If the individual's effort to change his own preferences is anything more than mere wishful thinking, then such effort will affect his choice pattern somewhere. In general, if the individual is trying to change his own preferences, then his behaviour in the present period would be different from what his behaviour would have been if he were choosing on the basis of his present preferences. After all, one of the main ways of changing our preferences or habits is to act as if we had a different set of preferences (or different 'habits') until such action comes naturally to us. Thus it is possible that while the individual may in fact have a sequence of preference orderings which he is successively trying to achieve over time, his behaviour in any given time period may not reveal his preference ordering (interpreted in the usual fashion) in that time period. Therefore not only Majumdar's rational individual may say, "I know what I should do, but I have no wish to do it because I have not prepared myself for it – yet" [see Majumdar (1980)] but he may also say, "I know what I want but I cannot choose what my present desires dictate since I have set before myself the goal of desiring in the future, things different from what I desire now." Thus Majumdar's analysis points to another interesting reason why an individual's choices, though consistent in some sense, may not reveal his preferences.

Bibliography

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