Analyse & Kritik 29/2007 (© Lucius & Lucius, Stuttgart) p. 226–234

Marc Fleurbaey

Living Standards and Capabilities: Equal Values or Equal Sets?*

Abstract: Inspired by Gaertner and Xu (2006), this paper examines the possibility to construct a social ordering over distributions of capability sets, and a measure of the value of individual capability sets, such that perfect equality of sets, across individuals, is preferable to a simple equality of the value of sets. It is shown that this is a rather demanding requirement.

0. Introduction

Gaertner and Xu (2006) propose an interesting way to measure the ethical value of capability sets. Capability sets, as is well known, are opportunity sets giving access to vectors of functionings, and functionings are all kinds of personal achievements that individuals may obtain in their life, in the terminology proposed by Sen.[1] In Sen's theory of justice, social institutions should seek equality of capabilities across the population, or at least give priority to the persons with the worst situations as measured in terms of capabilities. Sen has not proposed a precise method to evaluate and compare capability sets, focusing on the general idea that capabilities are the appropriate framework for interpersonal comparisons rather than on the specific ways in which this idea could be implemented.

Some rough applications of the capability concept have been made, the most famous being the Index of Human Development computed by the United Nations Development Program. This particular index, however, is only very loosely connected to the theory, especially because it is a simple average of aggregate indexes of income, life expectancy and education. It does not involve at any stage of its computation the idea that the capability set should be measured at the individual level before any computation of a population-wise index. There is also a wide empirical literature which studies how individuals feel free and able to do various things[2], but very little theoretical effort to elaborate a rigorous measure of the value of a capability set.

In this context, Gaertner and Xu's measure is a valuable contribution. It can be presented, with some simplification, as follows. In Figure 1 the thick

* This paper has benefited from very helpful comments by M. Ahlert and two anonymous referees.

[1] See, e. g., Sen 1992.

[2] See e.g. Anand et al. 2005; Kuklys 2005.

line delineates a capability set in a two-dimensional space (corresponding to two dimensions of functionings). The ethical value of this set is measured by the smallest distance of the boundary to the origin (Gaertner and Xu allow for a reference point that may differ from the origin), which is also the radius of the greatest circle that stands below the upper boundary of the set.

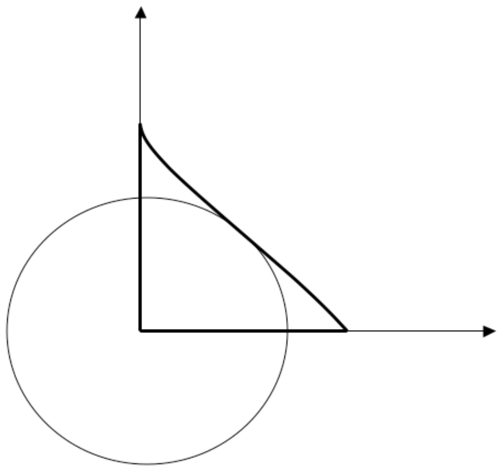

Figure 1: Gaertner and Xu's value of a set

This measure is nondecreasing with respect to set inclusion and satisfies other interesting properties listed by Gaertner and Xu. It has, however, two features which may be considered disadvantageous. It is not strictly increasing, as illustrated in Figure 2. It is also possible to find sets of equal value which differ greatly, as shown in Figure 3.

Why are such considerations relevant? If the functionings are measured in a way that makes more achievement in any dimension a good thing, it seems that expanding a capability set in any direction is a strict improvement and should be recorded as such. For instance, if it is possible to obtain more safety without any sacrifice on other dimensions such as income, health or education, the capability set appears better. Similarly, if more health is accessible, more education, or higher income, one would like to always record this as a strict improvement. Therefore, it appears desirable that the measure of the value of a capability set be strictly increasing with respect to strict set inclusion.

Insisting on equality of capability sets rather than equality of their values may seem less compelling. After all, if two sets have the same value, why bother if they are not exactly identical? It is obvious for instance that men and women will never have access to the same functionings, so that it appears more sensible to seek to give them capability sets of equal values. Moreover, the fact that capability sets are multi-dimensional makes it impossible, in the usual setting with continuous measures of achievement in the various functionings, to find any numerical measure that would give a different value to each set. Some kind of

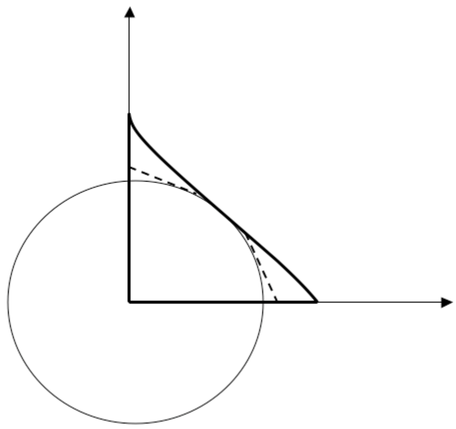

Figure 2: Equal value for strictly included sets

lexicographic ordering of sets would certainly be conceivable, but any sensible measure will give the same value to very different sets. That being said, the idea that individuals should, whenever possible (and efficient), be given identical sets rather than sets of equal value retains some attraction. For instance, individuals may have various personal preferences and even when they have sets of equal value, they may consider that the sets are not equivalent in their eyes. No such problem can occur if their sets are identical: we are then sure that, independently of their preferences, they consider them equivalent. In addition, it is clear that we want human beings to have access to the same variety of life options. For instance, it would be regrettable if some individuals only had access to manual jobs and others only to intellectual jobs, even if things were arranged so that the two arrays of possibilities had the same value.

Can the Gaertner-Xu measure be refined in this direction? It is not difficult to construct a measure of capability sets that is strictly monotonic. What is more interesting is that it is possible to associate such a measure to a social ordering over distributions of capability sets (a distribution is a list of the sets allotted to the population) such that a perfect equality of capability sets is the optimal distribution whenever possible and efficient. This can be obtained in spite of the fact that the value measure gives the same value to sets which are very different. Strict monotonicity of the measure, interestingly, is a key ingredient in this solution. There are, however, obstacles on the road and this paper shows that some natural requirements expressing preference for equalization of sets (not just of values of sets) are impossible to satisfy. Preference for equality of sets is definitely a demanding property.

The paper is structured as follows. The framework and the main definitions are introduced in Section 1. Then it is shown in Section 2 that a paradoxical impossibility mars the project of formalizing preference for perfect equality of sets.

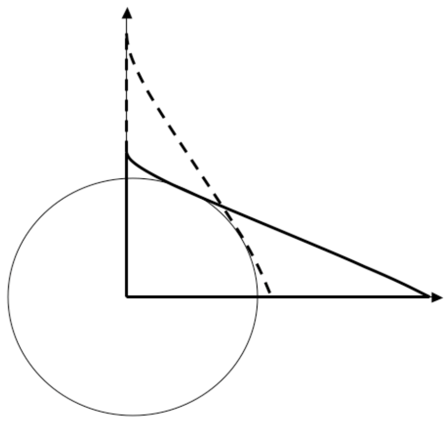

Figure 3: Equal value for very different sets

In Section 3, it is further explained how such preference for equality requires a strong kind of non-separability across individuals in the evaluation of distributions of capability sets. A more positive result is provided in Section 4, with an example of a social orderings that prefers perfect equality of sets. Section 5 offers concluding remarks.

1. The Framework

The model is similar to that retained in Gaertner and Xu (2006), with some simplifications. In particular, we assume that there are only two dimensions of functionings, and the analysis can be extended in a straightforward way to more dimensions. A capability set is assumed to be a regular comprehensive subset of $\Re^2_+$. A regular set is equal to the closure of its interior (this condition excludes sets made of segments of lines). A comprehensive set contains all the points below each of its points. Let C denote the set of regular comprehensive subsets of $\Re^2_+$.

The population has n individuals. A capability profile is denoted $C = (C_1, \ldots, C_n)$, where $C_i \in \mathrm{C}$ for all i=1,. . . ,n.

Our problem is twofold. We want to be able to rank capability sets with a value measure, and to rank capability profiles with a complete ordering over C^n. These two objects are naturally linked in the following way. The ordering over C^n (capability profiles), which will be denoted R, induces a complete ordering $\succeq$ over C, i.e. a ranking of capability sets, by the condition

$$C \succeq C' \Leftrightarrow (C, \ldots, C)\, R\, (C', \ldots, C').$$

In other words, a set is better than another if giving it to everybody is better than giving the other set to everybody. We will not be especially interested in a numerical representation of the ordering $\succeq$ in the first sections of this paper.

A profile $C = (C_1, \dots, C_n)$ is called "egalitarian" if $C_i \approx C_j$ for all i,j and "perfectly egalitarian" if $C_i = C_j$ for all i,j. Informally, an egalitarian profile has sets of equal value, whereas a perfectly egalitarian profile has exactly identical sets.

2. A Paradox

Our main problem is to find an ordering R satisfying the following axiom, which says that it is not enough to equalize the value of capabilities, it is better to equalize the sets perfectly.

Perfect egalitarianism: For all egalitarian profiles $C = (C_1, \dots, C_n)$, if C is not perfectly egalitarian, then for all i=1,...,n,

$$(C_i, \dots, C_i) \, P \, (C_1, \dots, C_n) \, .$$

This requirement says that if, in a given profile, all individuals sets have the same value but some of them differ, it would be better to give any of them to everyone.

One could object to this ethical ideal that, as it was mentioned in the introduction, it is in practice impossible to give exactly the same menu of options to different individuals. But the definition of the ideal should not be affected by this kind of feasibility constraint. Even if different individuals (such as men and women) will necessarily have different menus, it may be valuable to seek to reduce the differences in contents between their menus. Perfect egalitarianism is a property of the ordering, and once the ordering is defined, it is applied to whatever set of feasible allocations is available.

This condition is not easy to handle, because it involves the notion of egalitarian profile, which is itself induced by the value ranking $\succeq$ associated to R and is not explicit. Moreover, this condition does not say anything about non-egalitarian profiles. It would therefore be convenient to have a property applying to a broader range of cases, and the satisfaction of which would be easier to check.

Here is a natural candidate. It says that whenever two sets dominate each other in an area delimited by a cone, it is an improvement to reduce the gap between them within the cone, while leaving the rest unchanged. (A cone is a set such that if x belongs to it, so does αx for all $\alpha \geq 0$.) Its name is inspired from Hammond's (1976) equity condition. Let the symbol $\subset$ denote strict set inclusion.

Perfect equity: For all profiles $C = (C_1, \dots, C_n)$, $C' = (C'_1, \dots, C'_n)$, all i,j, if $C'_k = C_k$ for all $k \neq i, j$, and if there is a cone G such that

$$(G \cap C_j) \subset \left(G \cap C'_j\right) \subset (G \cap C'_i) \subseteq (G \cap C_i) \, ,$$

while for the complement cone H, $(H \cap C_j) = \left(H \cap C'_j\right)$ and $(H \cap C'_i) = (H \cap C_i)$, then $C'PC$.

This is illustrated in Figure 4, for the case of two agents. The sets modified as indicated by the dotted curves are closer than the initial sets, and Perfect equity says that this change is an improvement.

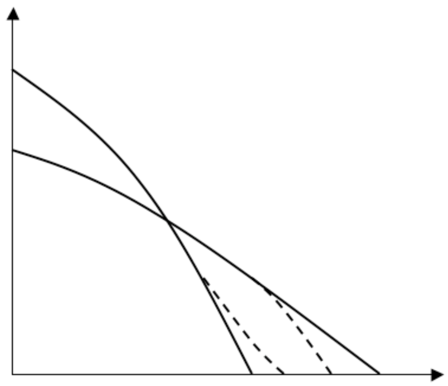

Figure 4: Illustration of Perfect Equity

The problem is that Perfect equity is not satisfied by any ordering, because it is self-contradictory. This is shown in Figure 5, with an example involving four agents. In the figure, the sets B and C are modified as indicated by the northwestern dotted line, which becomes their common boundary, and similarly A and D acquire the southeastern dotted line as their common boundary. By Perfect equity, the two changes are improvements. But these changes can be decomposed in a different way. Taking the whole orthant as the relevant cone, one sees that the inequality between A and B is increased, and similarly for C and D. Therefore Perfect equity concludes that the new situation is worse than the initial situation. This is a contradiction.

Observe that a domain restriction, such as requiring all sets to be convex, would not help very much unless it is so drastic that it prevents set boundaries to cross.

One might think of avoiding this problem by reducing the application of Perfect equity to cases when the whole orthant is taken as the relevant cone. This means that it would apply only to sets which are in a situation of total domination (no crossing of boundaries), but this weak version of Perfect equity is not strong enough to imply Perfect egalitarianism, which is precisely about making sets equal even when their boundaries cross.

The above impossibility, however, does not arise when the population is limited to two agents. In this case, the family of orderings (inspired from Herrero et al. 1998) which rank capability profiles on the basis of the intersection set satisfy both Perfect equity and (therefore) Perfect egalitarianism:

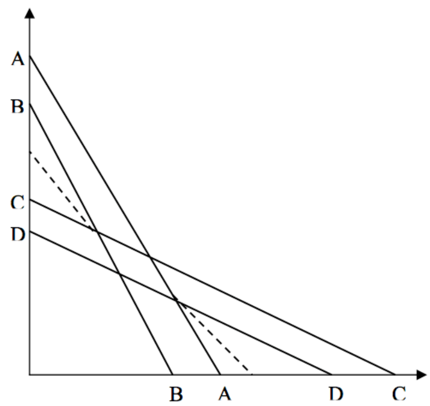

Figure 5: Paradox of Perfect Equity

Common set ordering: For all profiles $C = (C_1, C_2)$, $C' = (C'_1, C'_2)$, CRC' if and only if

$$V(C_1 \cap C_2) \geq V(C'_1 \cap C'_2),$$

where V is a strictly increasing (with respect to inclusion) real-valued function on C.

3. Separability

Consider the family of "individualistic" orderings, defined by the condition

$$(V(C_1), \ldots, V(C_n)) R^* (V(C'_1), \ldots, V(C'_n)),$$

where R^* is an ordering over $\Re^n$, and V is some increasing function. It is obvious that none of these orderings satisfies Perfect egalitarianism. Moreover, the failure to find a simple condition like Perfect equity suggests that Perfect egalitarianism requires a strong degree of non-separability in the ordering.

For any $G \subset \{1, \ldots, n\}$, let C_G denote the subprofile $(C_i)_{i \in G}$ and C_{-G} the subprofile $(C_i)_{i \notin G}$.

Separability: For all $G \subset \{1, \ldots, n\}$, all $C, C' \in \mathrm{C}^n$,

$$(C_G, C_{-G}) R (C'_G, C_{-G}) \Leftrightarrow \left(C_G, C'_{-G}\right) R \left(C'_G, C'_{-G}\right).$$

Perfect egalitarianism is incompatible with Separability. This can be seen with three agents, and the argument can be immediately generalized to more than three agents. Let A,B be two sets from C, which have equal value but are different: $A \approx B$, $A \neq B$. The profile (A,A,B) is better than the profile (A,B,B) because by Perfect egalitarianism the profile (A,A,A) is better than

(A,B,A), and by Separability the third set can be replaced by B without altering the ranking. But by a symmetrical argument the profile (A,B,B) is better than (A,A,B), which is contradictory.

4. Perfect Egalitarianism

The previous sections show that Perfect egalitarianism is a rather demanding condition in terms of non-separability. We end this short analysis by exhibiting an example of a family of orderings which satisfy this axiom.

Observe that the Common set ordering, which is easily generalized to more than two agents by simply taking the intersection $C_1 \cap \ldots \cap C_n$, satisfies Perfect egalitarianism. Its obvious drawback, though, is that it completely neglects the bigger sets and therefore fails to satisfy a basic Paretian monotonicity condition (i.e. when a set is expanded for one individual, the profile is better). Just as the leximin criterion refines the maximin criterion, one can refine the Common set ordering in order to alleviate this problem. Let $\geq_{lex}$ denote the leximin criterion, i.e. the smallest component of the left-hand vector is greater, or in case of equality the second smallest component is greater, and so on.

Common set leximin ordering: For all profiles $C = (C_1, \ldots, C_n)$, $C' = (C'_1, \ldots, C'_n)$, CRC' if and only if

$$\left(V\left(\bigcap_{i \in G} C_i \right) \right)_{G \subseteq \{1,\ldots,n\}} \geq_{lex} \left(V\left(\bigcap_{i \in G} C_i{}' \right) \right)_{G \subseteq \{1,\ldots,n\}},$$

where V is a strictly increasing real-valued function on C.

It is essential, for this ordering to satisfy Perfect egalitarianism, that the value function V be strictly increasing. Indeed, if two sets $A \subset B$ have the same value, i.e. $V(A) = V(B)$, then the profile (A,B) is considered equivalent to the profile (B,B) by Common set leximin ordering, in violation of Perfect egalitarianism.

This suggests that it may be important, when one wants to measure the value of capability sets, to bear in mind the application of such a measure for the social evaluation of distributions of capability sets.

5. Conclusion

If one considers it valuable to provide individuals not only with capability sets of equivalent values, but of exactly identical content, it is not easy to devise a social ordering over capability profiles, and a measure of set value, which embodies this judgment. The social ordering must be strongly non-separable and the value function must be strictly increasing with respect to set inclusion.

It appears a natural research program to seek new and interesting measures of living standards. The above analysis suggests that it may be useful to keep this program connected to the broader project of constructing orderings for the distribution of living standard across the population. From a good social ordering over capability profiles one necessarily derives an interesting measure of living standards, and the requirements for a good social ordering may imply some interesting requirements (such as strict monotonicity) for the measure of living standards.

Bibliography

Anand, P./G. Hunter/I. Carter/K. Dowding/F. Guala/M. van Hees (2005), *The Development of Capability Indicators and Their Relation to Life Satisfaction*, Milton Keynes, Economics Department, The Open University

Gaertner, W./Y. Xu (2006), Capability Sets as the Basis of a New Measure of Human Development, in: *Journal of Human Development 7*, 311–321

Hammond, P. J. (1976), *Equity, Arrow's Conditions and Rawls' Difference Principle*, in: *Econometrica 44*, 793–804

Herrero, C./I. Iturbe-Ormaetxe/J. Nieto (1998), Ranking Opportunity Profiles on the Basis of the Common Opportunities, in: *Mathematical Social Sciences 35*, 273–289

Kuklys, W. (2005), *Amartya Sen's Capability Approach: Theoretical Insights and Empirical Applications*, Berlin

Sen, A. (1992), *Inequality Re-examined*, Oxford