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Choices, Norms and Preference Revelation*

Abstract: This paper considers lexical combinations of choice functions where at least one is interpreted as arising from a norm. It is shown that in for all possibilities in which a norm is present, in general final choice may be consistent with preference optimization, but that it need not be so. It is concluded therefore that a fruitful approach to understanding the effect of norms on choice is to consider particular classes of norms rather than norms in general as in the work by Wulf Gaertner among others.

0. Introduction

The purpose of this paper is to consider the extent to which there are clear cases where it is possible to say whether a preference is revealed or not by choice behavior that depends in a certain way on a norm as well as possibly on preference optimization. It will be shown that if a norm is present then, given the way choices are induced in this paper, final choice may or may not be consistent with preference optimization.

The significance of this is twofold. First, in economics preference revealed by choice has long been regarded as the indicator of both individual wellbeing and as the relevant information for the choice of public policies. Second, preferences revealed by choices are the main source of testable restrictions on choices since they must then have some strong properties.

The approach in this paper is to formulate choice as a function of both preference optimization and a norm. Preference optimization simply requires that, from any possible set of alternatives, the highest ranked may be chosen. This requires no further explanation since this approach almost completely pervades the theory of rational choice. Norms on the other hand do require some explanation and the same is true for the way norms and preference optimization may jointly determine choice behavior.

Norms, especially those relating to choice behavior, are taken to be rules of the kind “must do”, “may do” or “must not do”. Of course, these are not logically independent, given the standard logical connectives. For example, “must not do” is simply the negation of “may do”. Norms are discussed further in the next section.

That leaves the function that “aggregates” a preference and a norm into a

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choice. In this paper, the structure of this function is lexical in that it assigns priorities and precludes tradeoffs. That is, chosen alternatives are selected in two stages and there are two possibilities depending on priorities in a way that will be made clear.

However, limiting attention to lexically determined choices might seem to neglect an important possibility that is at least implicitly assumed in the standard theory of rational choice. This is that agents trade-off qualitatively different kinds of things such as preferences and norms into an all things considered preference. However, tradeoffs require that whatever is traded is measurable in a way that permits comparisons. Since many considerations are only ordinally measurable such comparisons will not be possible and aggregation according to priorities is then reasonable. See for example, Sen (1993), Baigent and Gaertner (1996), Baigent (1995) and Manzini and Mariotti (2005). Furthermore, in the finite framework of this paper, the use of the familiar continuity property usually assumed for preferences over continuous spaces of alternatives and which rules out lexicographic preferences is not available. Given that rationality does not require such trade-offs, to consider them in this paper would beg the very question that is central to the paper. It is for this reason that only lexical aggregation is considered.

The structure of the paper is as follows. Section 1 gives an informal discussion of the approach followed in later sections. Concepts are formulated in section 2 and used in the results given in section 3. Section 4 concludes.

1. Norms and Preference Optimization

The key features of norms that are required for this paper may be introduced by an example given in Sen (1993). Let an agent choose y from the set {x, y, z} and z from the set {y, z}, written $C(\{x, y, z\}) = \{y\}$ and $C(\{y, z\}) = \{z\}$ respectively. This is not consistent with the maximization of any preference. To see this note that y rejected from {y, z} requires z strictly preferred to y and this is inconsistent with choosing y and rejecting z from {x, y, z} if choices are induced by maximizing a preference. Since there is no preference ranking that simultaneously has opposite strict preferences on any pair of alternatives, these choices are inconsistent with preference optimization.

However, Sen has offered the following interpretation. Let x, y and z denote three possible pieces of a homogeneous cake that differ only in size with x being the largest and z being the smallest. Furthermore, let an agent prefer larger to smaller pieces of cake. Finally, assume that the agent wishes to be polite where this requires never choosing the largest piece of cake. Such an agent is not obviously irrational in any sense even though behavior is not consistent with preference optimization.

It will be useful to consider this agent's choices as involving two steps. In the first step a politeness norm, N, determines which pieces of cake may be chosen without being impolite. In this example, y and z may be chosen from {x, y, z} without being impolite and this may be written $C_N(\{x, y, z\}) = \{y, z\}$.

Then in the second step, the preference, P, for larger over smaller pieces of cake determines the choice from {y, z} and this may be written, $C_P(\{y, z\}) = \{z\}$. Thus, the final choice $C(\{x, y, z\}) = \{y\}$ is the composition of the two functions C_N and C_P, so that $C(\{x, y, z\}) = C_P\,(C_N(\{x, y, z\})) = C_P(\{y, z\}) = \{y\}$. Similarly, $C(\{y, z\}) = C_P\,(C_N(\{y, z\})) = C_P(\{z\}) = \{z\}$. In this interpretation of Sen's example, the role of the norm is that of a self-imposed constraint on preference optimization.

For the purposes of this paper, it is important to be clear that the norm is taken to be an internalized norm. That is, it is part of the characterization of the agent's identity according to which, an agent is a triple consisting of a preference, a norm and a function that induces choices from a preference and a norm. In the example, the function is the composition of the two functions that represent the norm and preference respectively.

This contrasts with a norm that is not internalized in which case the norm is not part of the agent's identity. For example, in repeated interactions the behavior of agents may conform to a norm even though they are characterized in the usual way merely be a preference. It is interesting that in such repeated interactions, cooperative behavior norms may be satisfied in some equilibria. Roughly speaking however, in such cases behavior obeys a norm in order to avoid reactions by others while behavior required by an internalized norm is independent of reactions by others. In those cases behavior implements an agent's identity for which norms are constitutive.

Just as the norm is given priority over preference optimization in the interpretation given of Sen's example, it is also possible to consider cases in which this priority is reversed. For example, suppose now that x and y are pieces of cake that are the same size, both of which are larger than piece z. A larger cake preferring agent will now be indifferent between x and y, both of which will be preferred to z. Assume also that politeness is now taken to involve choosing the piece closest to the agent on the plate when it is offered. If the agent gives priority to preference optimization over politeness, and a plate with x, y and z on it is offered then the agent will choose whichever of x and y is the nearer. However, if only x and y are offered, and if the one chosen when all three are available is now further away, it will not be chosen. Again, the final choice behavior is not consistent with the optimization of any preference.

Finally, note that a norm may be consistent with the optimization of some ranking of the alternatives. For example, choose whatever will "maximize the welfare of your children" will be consistent with preference optimization whenever the concept of childrens' welfare is representable by a ranking. Thus, in this paper, choice is considered to be induced by compositions of functions each of which may or may not be consistent with preference optimization. All cases are considered with attention given to the possibilities for different priorities between norm satisfaction and preference optimization.

2. Definitions and Concepts

For all sets S, $|S|$ denotes its cardinality. X, $2 < |X| < \infty$, denotes a set of **alternatives**. and $\mathscr{X} = 2^X \backslash \emptyset$ is the set of all non empty subsets of X. R denotes a **binary relation on $\boldsymbol{X}$** ($R \subseteq X \times X$). A binary relation R on X is a **weak order** iff it is **complete** and **transitive**. That is, iff:

$$((\forall x, y \in X)[(x,y) \in R \vee (y,x) \in R]) \text{ and}$$

$$((\forall x, y, z \in X)\,[((x,y) \in R \wedge (y,z) \in R) \Rightarrow (x,z) \in R])$$

respectively.

For all weak orders R on X, and all $S \in \mathscr{X}$, $G(S,R) = \{x \in S : (\forall y \in S)(x,y) \in R\}$ is the non empty subset of ***R*-greatest** alternatives in S. A **choice function** on X is a function $C : \mathscr{X} \to \mathscr{X}$ that assigns a non empty subset $C(S)$ of S to all non empty subsets S of X. $\mathscr{C}$ denotes the set of all choice functions.

For all $C \in \mathscr{C}$, C is **consistent with optimizing a weak order** (COWO) if and only if there is a weak order R on X such that, for all $S \in \mathscr{X}$, $C(S) = G(S,R)$. Let $\mathscr{C}_W$ denote the set of all COWO choice functions. For all weak orders R_i on $\mathscr{X}$, C^i will denote the choice function such that for all $S \in X$, $C^i(S) = G(S, R_i)$.

The following two properties are imposed on choice functions to exclude uninteresting cases.

For all $C \in \mathscr{C}$, C is **relevant** if and only if, for some $S \in \mathscr{X}$, $C(S) \neq S$.

For all $C \in \mathscr{C}$, C is **permissive** if and only if, for some $S \in \mathscr{X}$, $|C(S)| > 1$.

An irrelevant choice function would not reject any alternatives from any subset. Choice functions that are not permissive completely determine final choices irrespective of other considerations of preference optimization or norm satisfaction.

For all $C \in \mathscr{C}$, C has **property** α if and only if, for all $S, T \in \mathscr{X}$ such that $S \subseteq T$ and all $x \in X$: $x \in S \cap C(T)$ implies $x \in C(S)$.

For all $C \in \mathscr{C}$, C has **property** γ if and only if, for all $S, T \in \mathscr{X}$ and all $x \in X$: $x \in C(S)$ and $x \in C(T)$ implies $x \in C(S \cup T)$.

$C \in \mathscr{C}$ has both properties α and γ if and only if it is COWO. See Sen (1970; 1986).

$\bar{\mathscr{C}}_{\alpha\gamma}$ will denote the set of choice functions that either do not have property α or do not have property γ. Equivalently, $\bar{\mathscr{C}}_{\alpha\gamma} = \mathscr{C} \backslash \mathscr{C}_W$.

For all $C_i, C_j \in \mathscr{C}$, the composition $C_i \circ C_j$ of C_i and C_j is the choice function such that, for all $S \in \mathscr{X}$, $C_i \circ C_j(S) = C_i(C_j(S))$.

With some abuse of notation, $\mathscr{C}_W \circ \bar{\mathscr{C}}_{\alpha\gamma}$ denotes the set of all choice functions that can be obtained by a composition, $C = C_i \circ C_j$, of a COWO choice function $C_i \in \mathscr{C}_W$ and a choice function $C_j \in \bar{\mathscr{C}}_{\alpha\gamma}$ that is not COWO. That is:

$$\mathscr{C}_W \circ \bar{\mathscr{C}}_{\alpha\gamma} = \left\{ C \in \mathscr{C} : \left(\exists (C_i, C_j) \in \mathscr{C}_W \times \bar{\mathscr{C}}_{\alpha\gamma}\right) C_i \circ C_j = C \right\}$$

Similarly, $\bar{\mathscr{C}}_{\alpha\gamma} \circ \mathscr{C}_W$ denotes the set of all choice functions that can be obtained by a composition, $C = C_i \circ C_j$, of a choice function $C_i \in \bar{\mathscr{C}}_{\alpha\gamma}$ that is not COWO with a choice function $C_j \in \mathscr{C}_W$ that is COWO. That is:

$$\bar{\mathscr{C}_{\alpha\gamma}} \circ \mathscr{C}_W = \left\{C \in \mathscr{C} : \left(\exists (C_i, C_j) \in \bar{\mathscr{C}_{\alpha\gamma}} \times \mathscr{C}_W\right) C_i \circ C_j = C\right\}.$$

$\mathscr{C}_W \circ \mathscr{C}_W$ and $\bar{\mathscr{C}_{\alpha\gamma}} \circ \bar{\mathscr{C}_{\alpha\gamma}}$ are defined analogously.

For all weak orders R_i and R_j on X, L_{ij} will denote the weak order on X such that, for all $x, y \in X$, $xL_{ij}y$ if and only if either xP_jy or $(xI_jy \wedge xR_iy)$. L_{ij} will be called the lexicographic extension of R_i and R_j.

3. Results

The first result is offered for completeness and is intended as to provide a contrast for the cases considered in the results that follow it. It shows that all choice functions that are obtained by the composition of choice functions that are both COWO are also COWO.

Lemma *If $C^i, C^j \in \mathscr{C}_W$ then, for all $S \in \mathscr{X}, C^i(C^j(S)) = G(S, L_{ij})$.*
Proof of Lemma: If $C^i, C^j \in \mathscr{C}_W$ then, for all $S \in \mathscr{X}$, $C^i(C^j(S)) = G(G(S, R_j), R_i)$. Therefore, it must be shown that $G(S, L_{ij}) = G(G(S, R_j), R_i)$.

Assume that $x \in G(S, L_{ij})$. This implies that, for all $y \in S$, $xL_{ij}y$, and therefore either xP_jy or $(xI_jy \wedge xR_iy)$. In either case, xR_jy so that $x \in G(S, R_j)$. There are now two cases to consider. Firstly, if $G(S, R_j) = \{x\}$ then $G(G(S, R_j)R_i) = G(\{x\}, R_i) = \{x\}$ and $x \in G(G(S, R_j), R_i)$. Secondly, if $G(S, R_j) \neq \{x\}$, then for some $y \neq x$, $y \in G(S, R_j)$. It follows that xI_jy. Given that $x \in G(S, L_{ij})$ is assumed, and we now have $(xI_jy \wedge xR_iy)$, it follows that for all $y \in G(S, R_j)$, xR_iy and $x \in G(G(S, R_j), R_i)$.

Now assume that $x \in G(G(S, R_j), R_i)$. This implies that, for all $y \in S$, xR_jy and, for all $y \in G(S, R_j)$, xR_iy. There are now two cases to consider. Firstly, if $G(S, R_j) = \{x\}$ then, for all $y \in S$, xP_jy and therefore $xL_{ij}y$. This implies that $x \in G(S, L_{ij})$. Secondly, if $G(S, R_j) \neq \{x\}$, then for some $y \neq x$, $y \in G(S, R_j)$. Therefore, xI_jy and given that as shown earlier xR_iy, it now follows that $xL_{ij}y$ and $x \in G(S, L_{ij})$.

This, together with the fact that L_{ij} is a weak order gives us the first result.

Theorem 1 $\mathscr{C}_W \circ \mathscr{C}_W \subseteq \mathscr{C}_W$
Indeed, this result may easily be strengthened to $\mathscr{C}_W \circ \mathscr{C}_W = \mathscr{C}_W$.

The proofs of the results covering other cases use the following choice functions all of which are both relevant and permissive

(i) The choice function induced by maximizing the weak order in which x, y and z are all indifferent and all of which are strictly preferred to w. This is COWO by construction.

(ii) Any choice function for which $C(\{w, x, y, z\}) = \{x, y, z\}$, $C(\{x, y, z\}) = \{x, y\}$, $C(\{w, x, y\}) = \{w\}$ and $C(\{w, x, z\}) = \{z\}$. This violates α and γ.

Choice functions (i) and (ii) require that the cardinality of X is at least 4 and this is used in theorems 2 and 3.

(iii) $C(\{x, y, z\}) = \{x, y\}$, $C(\{x, y\}) = \{x, y\}$, $C(\{y, z\}) = \{y\}$ and $C(\{x, z\}) = \{x\}$. This is induced by maximizing the weak order for which x and y are indifferent and both are strictly preferred to z. This is COWO by construction.

(iv) $C(\{x, y, z\}) = \{x\}$, $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$ and $C(\{x, z\}) = \{z\}$. This violates α and is therefore not COWO.

(v) $C(\{x, y, z\}) = \{z\}$, $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$ and $C(\{x, z\}) = \{x, z\}$. This violates α and γ, and is therefore not COWO.

(vi) $C(\{x, y, z\}) = \{x\}$, $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$ and $C(\{x, z\}) = \{x\}$. This is induced by maximizing the weak order in which x is uniquely best and z is uniquely worst.

(vii) $C(\{x, y, z\}) = \{x, y, z\}$, $C(\{x, y\}) = \{x, y\}$, $C(\{y, z\}) = \{y, z\}$ and $C(\{x, z\}) = \{x\}$. This violates α and is therefore not COWO.

The next case to be considered is that in which preference optimization is given priority over norm fulfillment. Thus, the appropriate composition is taken from $\bar{\mathscr{C}}_{\alpha\gamma} \circ \mathscr{C}_W$. As the following result shows, such compositions may be either COWO or not COWO.

Theorem 2 $\left(\bar{\mathscr{C}}_{\alpha\gamma} \circ \mathscr{C}_W\right) \cap \bar{\mathscr{C}}_{\alpha\gamma} \neq \emptyset$ and $\left(\bar{\mathscr{C}}_{\alpha\gamma} \circ \mathscr{C}_W\right) \cap \mathscr{C}_W \neq \emptyset$.
Proof Let C_i be given by (ii) and let C_j be given by (i). Then $C_i\left(C_j(\{w, x, y, z\})\right) = \{x, y\}$ and $C_i\left(C_j(\{w, x, z\})\right) = \{z\}$. Thus, $C_i \circ C_j$ violates property α and is not COWO. This proves the first part of theorem 2. For the second part, let C_i be given by (v) and let C_j be given by (iii). Then $C_i\left(C_j(\{x, y, z\})\right) = C_i\left(C_j(\{x, y\})\right) = C_i\left(C_j(\{x, z\})\right) = \{x\}$, and $C_i\left(C_j(\{y, z\})\right) = \{y\}$. Therefore $C_i \circ C_j$ has properties α and γ so that it is COWO as required.

The next case is that in which the norm is given priority over preference optimization. Therefore, final choice behavior is induced by norm constrained preference optimization. Again, this composition may be either COWO or not COWO.

Theorem 3 $\left(\mathscr{C}_W \circ \bar{\mathscr{C}}_{\alpha\gamma}\right) \cap \bar{\mathscr{C}}_{\alpha\gamma} \neq \emptyset$ and $\left(\mathscr{C}_W \circ \bar{\mathscr{C}}_{\alpha\gamma}\right) \cap \mathscr{C}_W \neq \emptyset$.
Proof For the first part, let C_i be given by (i) and let C_j be given by (ii). Then, $C_i\left(C_j(\{w, x, y, z\})\right) = \{x, y\}$ and $C_i\left(C_j(\{w, x, z\})\right) = \{w\}$ and since this violates property α, $C_i \circ C_j$ is not COWO. For the second part, let C_i be given by (vi) and let C_j be given by (vii). Then, $C_i \circ C_j = C_i$ and $C_i \in \mathscr{C}_W$ is all that is required for this case.

Finally, even if final choice behavior depends only on two norms and not on preference optimization at all, final choice behavior may or may not be COWO.

Theorem 4 $\left(\bar{\mathscr{C}}_{\alpha\gamma} \circ \bar{\mathscr{C}}_{\alpha\gamma}\right) \cap \bar{\mathscr{C}}_{\alpha\gamma} \neq \emptyset$ and $\left(\bar{\mathscr{C}}_{\alpha\gamma} \circ \bar{\mathscr{C}}_{\alpha\gamma}\right) \cap \mathscr{C}_W \neq \emptyset$.
Proof For the first part, let C_i be given by (iv) and let C_j be given by (v). Then, $C_i\left(C_j(\{x, y, z\})\right) = \{z\}$ and $C_i\left(C_j(\{y, z\})\right) = \{y\}$. Since this violates property

α, $C_i \circ C_j$ is not COWO. For the second part, let C_i be given by (iv) and let C_j be given by (vii). Then, $C_i\left(C_j(\{x,y,z\})\right) = C_i\left(C_j(\{x,y\})\right) = \quad C_i\left(C_j(\{x,z\})\right) = \{x\}$, and $C_i\left(C_j(\{y,z\})\right) = \{y\}$. Therefore $C_i \circ C_j$ has both properties α and γ. It is therefore COWO.

4. Conclusions

While theorem 1 has shown that the combination of choices induced by preferences leads to choices that are consistent with optimization (theorem 1), in all other cases (theorems 2, 3 and 4) in which a norm is present, final choice may or may not reveal a preference. Whether the norm takes priority over preference optimization or vice versa, in general the final choice might reveal a preference but it need not do so. This even remains the case if choice is induced by combining two norms (theorem 4), neither of which is COWO.

No restrictions have been imposed on the choice functions arising from norms in this paper. That seems appropriate in general since norms are many and varied, and it seems unlikely that there are properties of choice functions that will be satisfied by all choice functions arising from norms. This suggests that work in which Wulf Gaertner has been heavily involved is a fruitful line of research in that it considers the analysis of particular classes of norms rather than norms in general. See Baigent and Gaertner (1995) and Gaertner and Xu (1997; 1999a; 1999b; 1999c; 2004). See also, Xu (2007).

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