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The Liberal Paradox

*Some Interpretations When Rights Are Represented As Game Forms**

Abstract: The paper seeks to interpret the liberal paradox in a framework where individual rights are represented as game forms. Several close counterparts, in this framework, of Sen's theorem are considered, and their intuitive significance is discussed.

1. Introduction

Sen's (1970a; b) paradox of the Paretian liberal,[1] which first formally demonstrated the tension between individual rights and welfaristic values, constitutes one of the central results in social choice theory and has had far reaching influence on the development of the subject. However, a number of writers[2] have argued that Sen's formal formulation of individual rights does not correspond to our intuition about rights, and that a more adequate formulation of rights can be given in terms of game forms that specify the permissible strategies of each individual in the society and also an outcome for each possible configuration of permissible individual strategies. In this paper, I shall not discuss the relative advantages and disadvantages of Sen's formulation of rights and the alternative formulation in terms of game forms, that Sen's critics have suggested. Instead, I shall focus on a different problem. Is it possible to have, in the game form approach to individual rights, an exact counterpart of Sen's theorem(s) on the impossibility of a Paretian liberal? The main purpose of this paper is to explore this issue.

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[1] The terms 'liberal' and 'libertarian' have been both used in the large literature that has developed around Sen's result. In this paper, I use these terms interchangeably.

[2] See, among others, Nozick 1974, Bernholz 1974, Gärdenfors 1981, Sugden 1985a, Gaertner, Pattanaik and Suzumura 1992, Seidl 1986, Suzumura 1990, Pattanaik 1994a; 1994b; 1995, and Pattanaik and Suzumura 1994a; 1994b.

The plan of the paper is as follows. I lay down the notation and basic definitions in Section 2. In Section 3, I give a brief statement of Sen's paradox. In Section 4.1, I summarize the main ideas underlying the game form formulation of individual rights. The rest of Section 4 is concerned with alternative translations of Sen's result in terms of the game form approach. I consider different versions of the paradox when individual rights are represented by game forms. Some of these versions turn out to be to be more interesting than the others. In Section 5, I summarize the main conclusions of the paper.

2. The Notation and Basic Concepts

Let X be the universal set of social alternatives and let $\boldsymbol{Z}$ be the set of all non-empty subsets of X. $\Re$ will denote the set of all orderings over X.[3] The elements of $\Re$ will be denoted by R, R' etc. and will be interpreted as weak preference relations ('at least as good as') over X. For all $x, y \in X$ and all $R \in \Re, xPy$ iff $[xRy$ and not $yRx]$. Thus, intuitively, P is the strict preference relation ('better than') corresponding to R.

Let $N = \{1, \ldots, n\}$ denote the society. A *social decision* rule (SDR) is a function $D : Z \times \Re^n \to \boldsymbol{Z}$ such that, for all $A \in \boldsymbol{Z}$ and all $(R_1, \ldots, R_n) \in \Re^n, D(A; R_1, \ldots, R_n) \subseteq \boldsymbol{A}$. $D(A; R_1, \ldots, R_n)$ is to be interpreted as the set of socially chosen alternatives when A is the feasible set of social alternatives and $(R_1, \ldots, R_n)$ is the profile of preference orderings of the individuals in the society.

A *game form* is an $(n+2)$-tuple $< A; S_1, \ldots, S_n; g >$, where: (1) $A \in \boldsymbol{Z}$ is the set of possible outcomes; (2) for every $i \in N$, S_i is the set of strategies of individual i; and (3) $g : \prod_{i \in N} S_i \to A$ is the outcome function which, for every n-tuple of strategies in $\prod_{i \in N} S_i$, specifies exactly one outcome in A. For all $A \in \boldsymbol{Z}$, a *game* is an $(N+1)$-tuple $< G; R_1, \ldots, R_n >$ where G is a game form and $(R_1, \ldots, R_n) \in \Re^n$.[4] The elements of $\prod_{i \in N} S_i$ will be denoted by $s = (s_1, \ldots, s_n)$, $s' = (s_1, \ldots, s_n)$ etc. For all $s \in \prod_{i \in N} S_i$, all $j \in N$, and all $s'_j \in S_j$, s/s'_j will denote $s^* \in \prod_{i \in N} S_i$ such that $s^*_i = s_i$ for all $i \in N - \{j\}$, and $s^*_j = s'_j$. For all $i \in N$, the elements of $\prod_{j \in N - \{i\}} S_j$ will be denoted by

[3] An ordering over X is a binary relation R that satisfies: (i) reflexivity: for all $x \in X, xRx$; (ii) connectedness: for all distinct $x, y \in X, xRy$ or yRx; and (iii) transitivity: for all distinct $x, y, z \in X, [xRy$ and $yRz]$ implies $[xRz]$.

[4] Strictly speaking, in defining a game, one should take the restrictions of $R_1, \ldots, R_n$ to the set of possible outcomes figuring in the game, rather than the orderings $R_1, \ldots, R_n$ which are defined over the universal set of alternatives, X. However, for the sake of economy in notation, I depart from the usual formal practice.

$s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$, $s'_{-i} = (s'_1, \ldots, s'_{i-1}, s'_{i+1}, \ldots, s'_n)$ and so on. For all $s_{-i} \in \prod_{j \in N-\{i\}} S_j$ and all $s'_i \in S_i$, (s_{-i}, s'_i) will denote s^* such that $s^*_j = s_j$ for all $j \in N - \{i\}$, and $s^*_i = s'_i$.

3. The Liberal Paradox

The first formulation (see Sen 1970 a; b) of the liberal paradox was in a framework where social preference, rather than social choice, was the primitive notion. Later, it was restated in terms of social choice (see, for example, Sen 1976; 1983). For our purpose, the social choice formulation will be most convenient. Accordingly, I consider the paradox in the framework of social decision rules which are based on the primitive notion of social choice as distinct from social preference.

I first define several well known properites of a GDR. Let D be a GDR. D satisfies:

Rejection (REJ): iff, for all A, $B \in \boldsymbol{Z}$, all $x \in X$, and all $(R_1, \ldots, R_n) \in \Re^n$, $[x \in B \subseteq A$ and $x \notin D(B; R_1, \ldots, R_n)]$ implies $[x \notin D(A; R_1, \ldots, R_n)]$;

Global Liberalism (GL): iff, for all $i \in N$,
there exist distinct $x, y \in X$ such that, for all $(R_1, \ldots, R_n) \in \Re^n$ and all $A \in \boldsymbol{Z}$, [if $x, y \in A$ and xP_iy, then $y \notin D(A; R_1, \ldots, R_n)$] and [if $x, y \in A$ and yP_ix, then $x \notin D(A; R_1, \ldots, R_n)$] ... (3.1);

Global Pareto Criterion (GPC): iff, for all $(R_1, \ldots, R_n) \in \Re^n$, all $A \in \boldsymbol{Z}$ and all $x, y \in X$, $[x, y \in A$ and xP_iy for all $i \in N]$ implies $[y \notin D(A; R_1, \ldots, R_n)]$.

Binary Liberalism (BL): iff, for all $i \in N$,
there exist distinct $x, y \in X$ such that, for all $(R_1, \ldots, R_n) \in \Re^n$, [if xP_iy, then $y \notin D(\{x, y\}, R_1, \ldots, R_n)$] and [if yP_ix, then $x \notin D(\{x, y\}; R_1, \ldots, R_n)$] ... (3.2);

Binary Pareto Criterion (BPC): iff, for all $(R_1, \ldots, R_n) \in \Re^n$ and all $x, y \in X$, $[xP_iy$ for all $i \in N]$ implies $[y \notin D(\{x, y\}; R_1, \ldots, R_n)]$.

These properites are very familiar in the literature on social choice theory and hardly need much explanation. It may, however, be worth noting the following points. First, Sen intended (3.1) and (3.2) to capture only some necessary conditions for an individual i to have a right, especially a right involving his autonomy over his private life (recall that the alternatives x and y, referred to in (3.1) and (3.2), were interpreted by Sen as social alternatives which were indentical in all respects except for some feature, say, i's religion, which related to the private life of i). Secondly, while GL assumes that (3.1) should hold for every i in N, Sen only required that (3.1) should hold for every i in some subset of N, containing at least two members. One can also weaken BL along similar lines. However, such weakening of GL and BL is not crucial for my purpose, and I shall use conditions GL and BL, though the weaker versions of these conditions were used by Sen in proving the liberal paradox. Lastly, it may be noted that BL and REJ, together, imply GL, though the converse is not necessarily true. Similarly, BPC and REJ, together, imply, but are not implied, by GPC.

The following two propositions represent two different versions of Sen's liberal paradox in the framework of a social decision rule.

Proposition 3.1: There does not exist any GDR that satisfies GL and GPC.

Proposition 3.2: There does not exist any GDR that satisfies BL, BPC and REJ.

The proofs of these propositions are well known and are omitted here.

4. Interpretation of the Liberal Paradox

In this section, I seek to interpret Propositions 3.1 and 3.2 in a framework where individual rights are represented by game forms.

4.1 Rights as Game Forms

As noted in the introduction, a number of writers have suggested formulations of rights that radically differ from Sen's formulation. While the alternative approaches suggested by these writers are similar in many ways, there are also significant differences.[5] In this paper, I shall focus on one of these formulations of individual rights, namely, the formulation in terms of (normal) game forms; I shall call it the game form approach to individual rights.

A game form representation of a rights-structure is simply a specification of a (normal) game form $G_A =< A; S_{1A}, \ldots, S_{nA}; g_A >$ for every $A \in \boldsymbol{Z}$, where, for every $i \in N, S_{iA}$ is interpreted as the set of all strategies *permissible* for individual i, given that A is the set of all feasible social alternatives. The game form $< A; S_{1A}, \ldots, S_{nA}; g_A >$ is intended to capture the powers, freedoms, immunities and claims conferred on the individuals by the rights structure,[6] when A is the set of all feasible social alternatives. A detailed

[5] For a discussion of some of these approaches, which are related but distinct from each other, see Gärdenfors 1981, Deb 1989; 1994, Hammond 1993; 1995, Pattanaik 1994b; 1995, and Peleg 1994.

[6] For an incisive discussion of the notions of power, freedom, immunity and claim, see Kanger and Kanger 1972.

discussion of the game from formulation of individual rights and how it differs from Sen's formulation is beyond the scope of this paper,[7] but, it is important to note the following difference between the two approaches. Sen's conception of an individual's right, as incorporated in GL and BL, involves restrictions on social choice which are contingent on the nature of individual preferences over certain pairs of social alternatives. In contrast, the game form formulation of individual rights does not refer at all to individual preferences over social alternatives; nor does it refer to the actual outcome of any game. The game form approach claims that, given a feasible set, A, of social alternatives, the substantive content of the rights that the individuals enjoy is entirely captured by the sets of permissible strategies of the individuals and the outcome function that specifies the outcome for each possible n-tuple of permissible strategies.

4.2 Interpretation of an SDR in the Game Form approach

To interpret Propositions 3.1 and 3.2 in the game form approach, we first need to interpret the notion of an SDR in terms of game forms and games. Suppose the rights-structure, in the sense in which the game form approach visualizes it, is given by a function $\boldsymbol{G}$ that specifies a game form G_A for every $A \in \boldsymbol{Z}$. Throughout the rest of this paper, $\boldsymbol{G}$ will denote the rights-structure in this sense. Given $\boldsymbol{G}$, for every $A \in \boldsymbol{Z}$ and every $(R_1, \ldots, R_n) \in \Re^n$, we have a game $(G_A; R_1, \ldots, R_n)$. I interpret the outcome of this game as the alternative that is 'socially chosen' when A is the set of all feasible social alternatives and $(R_1, \ldots, R_n)$ is the profile of individual orderings. The outcome of the game $(G_A; R_1, \ldots, R_n)$ will depend on how the individuals choose their respective strategies in the game$(G_A; R_1, \ldots, R_n)$. For our purpose, it is not necessary to have an explanation (possibly, based on a specific game theoretic notion of equilibrium) of how the individuals choose their strategies in different games of this type. It is enough to assume that, for every $A \in \boldsymbol{Z}$ and every $(R_1, \ldots, R_n) \in \Re^n$, they somehow finally choose their respective strategies in the game $(G_A; R_1, \ldots, R_n)$ so that there exists a function J which, for every $A \in \boldsymbol{Z}$ and every $(R_1, \ldots, R_n) \in \Re^n$, specifies exactly one n-tuple, $s^\star_A = (s^\star_{1A}, \ldots, s^\star_{nA})$, of strategies that the individuals actually choose in the game $(G_A; R_1, \ldots, R_n)$, where G_A is the game form specified by $\boldsymbol{G}$ for the set A. I write $s^\star_A = (s^\star_{1A}, \ldots, s^\star_{nA}) = J(G_A; R_1, \ldots, R_n)$, $s^\star_{iA} (i \in N)$ being the strategy chosen by individual i in the game $(G_A; R_1, \ldots, R_n)$. For all $A \in \boldsymbol{Z}$, and all $(R_1, \ldots, R_n) \in \Re^n$, the outcome of the game $(G_A; R_1, \ldots, R_n)$ is then given by $g_A(J(G_A; R_1, \ldots, R_n))$ (recall that g_A is the outcome finction in the game form G_A specified by $\boldsymbol{G}$ for A). We can now interpret an SDR D as a

[7] For such discussion, the reader may refer to Sugden 1985a, Gaertner, Pattanaik and Suzumura 1992, Pattanaik 1995, Riley 1989; 1990, and Sen 1992, among others.

function which, for every $A \in \boldsymbol{Z}$, and every $(R_1, \ldots, R_n) \in \Re^n$, specifies the (singleton) set $\{g_A(J(G_A; R_1, \ldots, R_n))\}$ as the set of socially chosen alternatives. Thus, under our interpretation of the SDR, the socially chosen outcome for a given $A \in \boldsymbol{Z}$ and a given $(R_1, \ldots, R_n) \varepsilon \Re^n$ is just the outcome of the game $(G_A; R_1, \ldots, R_n)$ as determined by whatever strategies the individuals may choose in the game and the outcome function g_A.

4.3 Re-interpretation of Proposition 3.1

First, I consider Proposition 3.1 and the two conditions, GPC and GL, which figure in it. Given the interpretation of an SDR in the game form framework, GL and GPC can be reformulated in an obvious fashion (I call these versions of GL and GPC, GL' and GPC', respectively):

GL': For all $i \in N$, there exist distinct $x, y \in X$ such that, for all $A \in \boldsymbol{Z}$, and all $(R_1, \ldots, R_n) \in \Re^n$, [if $x, y \in A$ and xP_iy, then $g_A(J(G_A; R_1, \ldots, R_n)) \neq y$] and [if $x, y \in A$ and yP_ix, then $g_A(J(G_A; R_1, \ldots, R_n)) \neq x$].

GPC': For all $A \in \boldsymbol{Z}$, all $(R_1, \ldots, R_n) \in \Re^n$ and all distinct $a, b \in X$, if [$a, b \in A$, and aP_jb for all $j \in N$], then $g_A(J(G_A; R_1, \ldots, R_n)) \neq b$.

GL' and GPC' constitute restrictions on the game forms specified by the rights structure $\boldsymbol{G}$, and the function J, taken together.

Proposition 4.1 below constitutes the counterpart of Proposition 3.1 in the game form approach to rights.

Proposition 4.1: There do not exist rights-structure $\boldsymbol{G}$ and function J such that GL' and GPC' are both satisfied.

Given the game form approach to individual rights, GL' cannot be interpreted as a condition that follows directly from the existence of rights (more specifically, from the existence of rights to autonomy in private affairs) for each individual. For, unlike Sen who interpreted (3.1) as a necessary condition for an individual i to have a right, the game from approach does not recognise any conceptual link between: (i) an individual's having a right; and (ii) that individual's preferences over different soical alternatives and the actual social choice of an alternative. It is only when we come to the issue of the exercise of rights that the game form approach recognises the relevance of individual preferences over social alternatives: given the powers, freedoms, claims and immunities which are guaranteed by the rights structure $\boldsymbol{G}$, the individuals' choices of strategies and, hence, the actual social outcome, will depend on individual preferences over the social alternatives. In view of this, it seems natural to regard GL' as a 'hybrid' condition that follows from: (i) the requirement that every individual should have rights as conceived in the game form framework; and (ii) certain empirical assumptions about how people behave when they have the powers, freedoms etc. guaranteed by the rights-structure $\boldsymbol{G}$. Thus, the following three intuitive statements, together, may be regarded as the conceptual basis of GL':

Every individual should have the right to autonomy in his private life. ... (4.1)

For all $i \in N$, if i enjoys the right to autonomy in his private life, then there exist distinct $x, y \in X$ such that, whenever x and y are both feasible, i has the power or ability to prevent y from being the social outcome, and also the power or ability to prevent x from being the social outcome. ... (4.2)

For all $i \in N$, if [two distinct social alternatives, a and b, are both feasible, i has the ability to prevent b from being the social outcome, and i strictly prefers a to b], then i will act in such a way that b will not be the outcome. ... (4.3)

The terms 'ability' and 'power' figuring in (4.2) and (4.3) are as yet undefined. They need to be given precise content and I shall presently take up this issue. However, the intuitive statements (4.1), (4.2) and (4.3) are helpful in understanding the significance of GL' in so far as GL' can be viewed as a consequence of (4.1), (4.2) and (4.3). One can then claim that the 'real' significance of Proposition 4.1 lies in the fact that it demonstrates the incompatibility of:

(i) the weak welfaristic judgement GPC',
(ii) the libertarian position (4.1);
(iii) (4.2) which reflects a specific intuition about what having rights to liberty in one's private life means; and
(iv) the behavioural assumption (4.3).

(4.2) and (4.3) involve the notion of i's ability or power, in a game form G_A, to prevent an alternative b from being the outcome. This intuitive notion can be given various precise interpretations. Consider two such interpretations given by (4.4) and (4.5) below, where it is assumed that G_A is the relevant game form and $a, b \in A$.

For all $s_{-iA} \in \prod_{j \in N - \{i\}} S_{jA}$, [there exists $\bar{s}_{iA} \in S_{iA}$ such that $g_A(s_{-iA}, \bar{s}_{iA}) \neq b$]. ... (4.4)

For all $s_A \in \prod_{j \in N} S_{jA}$, if $g_A(s_A) = b$, then there exists $\bar{s}_{iA} \in S_{iA}$ such that $g_A(s_A/\bar{s}_{iA}) = a$. ... (4.5)

If we use the interpretation of power given by (4.4), then we shall have the following versions of (4.2) and (4.3) respectively.

For all $i \in N$, if i enjoys the right to autonomy in his private life, then there exist distinct $x, y \in X$ with the following feature: for all $A \in \boldsymbol{Z}$ such that $x, y \in A$, and for all distinct $a, b \in \{x, y\}$, (4.4) holds.

... (4.2′)

For all $i \in N$, all $A \in \boldsymbol{Z}$ and all distinct $a, b \in A$, if (4.4) holds, then for all $(R_1, \ldots, R_n) \in \Re^n$ such that a $P_i b, g_A[J(G_A; R_1, \ldots, R_n)] \neq b$.
... (4.3′)

Replacing '(4.4)' by '(4.5)' in (4.2′) and (4.3′), we get versions of (4.2) and (4.3), which correspond to the notion of power given in (4.5) and which I shall call (4.2″) and (4.3″), respectively.

Both (4.2′) and (4.2″) seem to be plausible consequences of our intuition about the right to liberty in one's private life such as the right to choose one's own mode of worship, the right to eat vegetarian or non-vegetarian food and so on. What about the behavioural assumptions represented by (4.3′) and (4.3″)? (4.3′) is an implausible behavioral assumption. Let $N = \{1, 2\}$, $A = \{x, y, z, w\}$, and let the game form G_A be given as follow:

Figure 1: G_A

		2: s_{2A}	2: s'_{2A}
1	s_{1A}	x	y
1	s'_{1A}	z	w

It is easy to check that 1 has the ability, in the sense of (4.3′), to prevent w from being the outcome, and also the ability to prevent x from being the outcome. However, if (R_1, R_2) is such that $zP_1xP_1wP_1y$ and $yP_2xP_2wP_2z$, then the game $(G_A; R_1, R_2)$ will be the familiar game of prisoners' dilemma, and w will be the outcome of the game, assuming that the individuals will choose their respecitve dominant strategies (s'_{1A} for 1 and s'_{2A} for 2). Since xP_1w, this is clearly inconsistent with (4.3′).

(4.3″) is a more plausible assumption. Indeed, (4.3″) follows from (4.3‴) below:

For all $A \in \boldsymbol{Z}$, all $a \in A$ and all $(R_1, \ldots, R_n) \in \Re^n$, if (there does not exist $s_A \in S_A$ such that s_A is a Nash equilibrium in the game $[(G_A; R_1, \ldots, R_n)$ and $g_A(s_A) = a]$, then $[g_A[J(G_A; R_1, \ldots, R_n)] \neq a]$.
... (4.3‴)

Neither (4.3$''$) nor (4.3$'''$) can be possibly satisfied if, for some $A \in \boldsymbol{Z}$ and for some $(R_1, \ldots, R_n) \in \Re^n$, no Nash equilibrium (in pure strategies[8]) exists for the game $(G_A; R_1, \ldots, R_n)$ and no two distinct n-tuples of strategies in this game yield identical outcomes.

GL' can be regarded as a consequence of either (i) (4.1), (4.2$'$) and (4.3$'$), or (ii) (4.1), (4.2$''$) and (4.3$''$). The justification of GL' as a consequence of (4.1), (4.2$'$) and (4.3$'$) is not of much intuitive interest in view of the implausibility of (4.3$'$). I shall, therefore, ignore it, and focus on the justification of GL' as a consequence of (4.1), (4.2$''$) and (4.3$''$) and the corresponding interpretation of Proposition (4.1) as showing the logical incompaticility of GPC', (4.1), (4.2$''$) and (4.3$''$).[9] In assessing the intuitive significance of this incompatibility, several points may be noted. First, the incompatiblity, by itself, does not show an inconsistency between the welfaristic principle of GPC' and the libertarian position reflected in (4.1) and (4.2$''$). The tension arises from the presence of the behavioural assumption, (4.3$''$), in addition to GPC', (4.1) and (4.2$''$). In fact, following a line of reasoning owing to Sugden (1985b), it can be argued that it is not possible to prove a direct contradiction between GPC' and the libertarian position represented by (4.1) and (4.2$''$). This is because, while GPC' constitutes a restriction on the actual outcome of a game, depending on the individuals' preferences, (4.1) and (4.2) do not refer to either individual preferences or the actual outcome of any game.

Secondly, a difficulty in assessing the intuitive significance of the inconsistency of GPC', (4.1), (4.2$''$) and (4.3$''$) arises when, given the rights structure $\boldsymbol{G}$, it is impossible to satisfy (4.3$''$). Suppose the rights structure $\boldsymbol{G}$ is such that, for some $A \in \boldsymbol{Z}$, and for some $(R_1, \ldots, R_n) \in \Re^n$, the game $(G_A, R_1, \ldots, R_n)$ does not have any Nash equilibrium in pure strategies and no two distinct n-tuples of strategies in this game yield identical outcomes. Then (4.3$''$) cannot be satisfied, which will formally imply the failure to satisfy GPC', (4.1), (4.2$''$) and (4.3$''$), together. But this, in itself, does not reflect any conflict between different ethical values.[10] It only shows that, given the rights structure, the empirical assumption (4.3$''$) about how people act in games cannot be valid. Nor does the rights-structure $\boldsymbol{G}$ have to be very unintuitive for this possibility to arise. That, even with a very plausible rights-structure $\boldsymbol{G}$, there may not be any J satisfying either (4.3$''$) or (4.3$'''$) can be seen from the following example much discussed in the literature (see, for example, Gibbard 1974, Gaertner/Pattanaik/Suzumura 1992, Pattanaik

[8] Throughout this paper, I consider game forms and games with pure strategies only.

[9] In the early eighties, I had the opportunity of listening to a seminar presentation of Dr. P. Grout in England. As far as I can recall now, Dr. Grout interpreted Sen's liberal paradox in terms of a possible non-existence of a Nash-equilibrium belonging to the core. This interpretation is similar to the one that I am discussing here. I am, of course, happy to acknowledge the priority of Dr. Grout in this respect.

[10] Cf. Pattanaik 1995, and Deb, Pattanaik and Razzolini 1995.

1995). Let $N = \{1, 2\}$, and let the sole right under consideration be the right to choose one's own shirt. Let $X = \{ww, rr, wr, rw\}$, where wr denotes the social state in which 1 wears a white shirt and 2 wears a red shirt, ww denote the social state in which both individuals wear white shirts and so on (all aspects of the social states, other than the two individuals' shirts, are assumed to be fixed). Suppose G_X is as in Figure 2. (In Figure 2, w indicates the choice of a white shirt by the person under consideration and r denotes the choice of a red shirt; ww, wb etc. are the different social states as explained above.)

Figure 2: G_X

		2	
		w	r
1	w	ww	wr
	r	rw	rr

Intuitively, given the game form G_X, for each individual, it is feasible and permissible to choose either a white shirt or a red shirt, and each person gets to wear the shirt that he chooses to wear. Clearly, G_X corresponds to our intuition about the right to choose one's own shirt. Assume that (R_1, R_2) is such that $wwR_1rrR_1wrR_1rw$ and $wrR_2rwR_2rrR_2ww$. The game $(G_A; R_1, R_2)$ does not have a Nash equilibrium (in pure strategies), and, therefore, it is clear that there does not exist J that can possibly satisfy (4.3$''$) or (4.3$'''$) (recall that the function J determines the actually chosen strategies for the different games).

4.4 Re-interpretation of Proposition 3.2

I now consider Proposition 3.2. The reformulations of BL, BPC and REJ in the game form framework are given, respectively, by BL', BPC' and REJ' below.

BL': For all $i \in N$, there exist distinct $x, y \in X$ such that, for all $(R_1, \ldots, R_n) \in \Re^n$, [if xP_iy, then $g_{\{x,y\}}[J(G_{\{x,y\}}; R_1, \ldots, R_n)] \neq y$] and [if yP_ix, then $g_{\{x,y\}}[J(G_{\{x,y\}}; R_1, \ldots, R_n)] \neq x$].

BPC': For all $(R_1, \ldots, R_n) \in \Re^n$ and all distinct $a, b \in X$, if $[aP_jb$ for all $j \in N]$, then $[g_{\{a,b\}}[J(G_{\{a,b\}}; R_1, \ldots, R_n)] \neq b]$.

REJ': for all $A, B \in Z$, all $x \in X$ and all $(R_1, \ldots, R_n) \in \Re^n$, $[x \in B \subseteq A$ and $g_B[J(G_B; R_1, \ldots, R_n)] \neq x]$ implies $[g_A[J(G_A; R_1, \ldots, R_n)] \neq x]$.

Proposition 3.2 can be translated as follows:

Proposition 4.2: There do not exist rights-structure $\boldsymbol{G}$ and function J such that BL', BPC' and REJ' are all satisfied.

Since GL' follows from BL' and REJ', and GPC' follows from BPC' and REJ', Proposition 4.2 implies Proposition 4.1.

We can view BL' as a consequence of (4.1), and the intuitive assumptions (4.6) and (4.7) below:

For all $i \in N$, if i enjoys the right to autonomy in his private life, then there exist distinct $x, y \in X$ such that when x and y are the only feasible alternatives, i has the power to rule out y as the social outcome and also the power to rule out x as the social outcome. ... (4.6)

For all $i \in N$ and all distinct $a, b \in X$, if [a and b are the only two feasible alternatives, i has the ability to rule out b as the social outcome, and a P_ib], then b will not be the outcome. ... (4.7)

As in the case of (4.2) and (4.3), we need to interpret the notion of power figuring in (4.6) and (4.7). Given a feasible set A containing exactly two distinct alternatives, a and b, we can interpret i's power to rule out b as either (4.4) or (4.5) (note that when $|A| = 2$, (4.4) and (4.5) are equivalent). However, when there are exactly two feasible alternatives, x and y, which differ only with respect to i's private life, it seems very plausible to interpret the notion of power in (4.6) in a much stronger sense, namely, as the simultaneous existence of: (i) a permissible strategy of i that will ensure outcome x irrespective of what permissible strategies others adopt; and (ii) a permissible strategy of i that will ensure outcome y irrespective of the permissible strategies chosen by others. Accordingly, I consider the following precise versions of (4.6) and (4.7), respectively.

For all $i \in N$, if i enjoys the right to autonomy in his private life, then there exist distinct $x, y \in X$ for which the following holds: for all distinct $a, b \in \{x, y\}$, there exists $s_{i\{x,y\}} \in S_{i\{x,y\}}$ such that for all $s'_{-i\{x,y\}} \in \prod_{j \in N-\{i\}} S_{j\{x,y\}}$, $g_{\{x,y\}}(s'_{-i\{x,y\}}, s_{i\{x,y\}}) = a$. ... (4.8)

For all $i \in N$ and all distinct $a, b \in X$, if [there exists $s_{i\{a,b\}} \in S_{i\{a,b\}}$ such that for all $s'_{-i\{a,b\}} \in \prod_{j \in N-\{i\}} S_{j\{a,b\}}$, $g_{\{a,b\}}(s'_{-i\{a,b\}}, s_{i\{a,b\}}) = a$], then, for all $(R_1, \ldots, R_n) \in \Re^n$, $[aP_ib]$ implies $[g_{\{a,b\}}[J(G_{\{a,b\}}; R_1, \ldots, R_n)] = a]$. ... (4.9)

Since BL' can be regarded as a consequence of (4.1), (4.8) and (4.9), Proposition 4.2 (and hence, also Proposition 4.1, which follows from Proposition 4.2) can be interpreted as demonstrating the impossibility of simultaneously satisfying (4.1), (4.8), (4.9), BPC' and REJ'. Proposition 4.2, therefore, does not show a direct inconsistency between the libertarian position and welfarism: the tension demonstrated by Proposition 4.2 arises from the presence of the behavioural assumption (4.9) and the requirement of REJ' in addition to (4.1), (4.8) and BPC'.

(4.8) seems to conform to our intuition about the right to autonomy in one's private life and, therefore, (4.1) and (4.8), together, reflect a very plausible libertarian position in the game form approach. The plausibility of (4.9) is obvious, and BPC' is a very weak welfaristic restriction. This leaves us with REJ'. There are two alternative ways, one descrptive and the other normative, in which REJ' can be interpreted. Under the descriptive interpretation, REJ' is an assumption about how people behave. That is, REJ' is interpreted as saying that people choose their strategies in such a way that, for all $(R_1, \ldots, R_n) \in \Re^n$ and all $A, B \in \boldsymbol{Z}$, if $B \subseteq A$, then for all $x \in \boldsymbol{B}$, if x is not the outcome of the game $(G_B; R_1, \ldots, R_n)$, then x will not also be the outcome of the game $(G_A; R_1, \ldots, R_n)$. Interpreted in this fashion, REJ' is implausible: there does not seem to be any intuitive reason why people will behave in a way that will necessarily lead to the fulfillment of REJ'. A very convincing example due to Sugden (1985b) shows that REJ', regarded as an empirical assumption, can be violated even when the individuals behave in a way one would normally expect them to behave, and even when the game forms that represent the rights-structure $\boldsymbol{G}$ conform to our intution about some very familiar rights.

Alternatively, REJ' can also be interpreted as a normative requirement of social consistency. The condition that a feasible alternative that is socially rejected initially should not get socially chosen, when the feasible set of alternatives is expanded (individual preferences remaining the same), has an inherent attraction. It is possible to argue that this property should be satisfied by the social choices even if the procedure for arriving at social choices from different possible feasible sets is to let people freely choose their strategies in the respective game forms specified for these different sets. However, if one adopts such a normative interpretation of REJ', then it is worth noting that, even in the absence of any Paretian conditions and even when individuals act in a 'normal' fashion, REJ' can clash with our intuition about individual rights to liberty in private affairs. This is shown by the example of Sugden (1985b) referred to earlier (the example is as apposite for the normative interpretation of REJ' as for its interpretation as an empirical assumption). The point can also be illustrated by extending the example involving the choice of shirts, that I used at the end of Section 4.3. Continue to assume that G_X

is as already specified in that example. Further assume $G_{\{ww,rw\}}$, $G_{\{wr,rr\}}$, $G_{\{ww,wr\}}$ and $G_{\{rw,rr\}}$ to be as in Figure 3 below.

Like the earlier specification of the game form G_X in Figure 2, these additional specifications in Figure 3 are in conformity with our intuition about the right to choose one's own shirt. I continue to assume that $wwR_1rrR_1wrR_1rw$ and $wrR_2rwR_2rrR_2ww$. If (4.9) holds, then the outcomes of the games $(G_{\{ww,wr\}};R_1,R_2)$, $(G_{\{rw,rr\}};R_1,R_2)$, $(G_{\{ww,rw\}};R_1,R_2)$ and $(G_{\{wr,rr\}};R_1,R_2)$ must be wr, rw, ww and rr, respectively. It is then clear that, whatever be the outcome of the game $(G_X;R_1,R_2)$ (and there has to be an outcome for this game), REJ' must be violated.

Figure 3

$G_{\{ww,rw\}}$

		2: w
1	w	ww
	r	rw

$G_{\{wr,rr\}}$

		2: r
1	w	wr
	r	rr

$G_{\{ww,wr\}}$

		2: w	2: r
1	w	ww	wr

$G_{\{rw,rr\}}$

		2: w	2: r
1	r	rw	rr

5. Concluding Remarks

In this paper, I have sought to interpret Sen's liberal paradox in a framework where the structure of rights is given by the specification of a game form for each possible feasible set of alternatives. The main conclusions of my discussion may be summarized as follows:

(1) In the game form framework, the paradox can be interpreted in at least two plausible ways. First, we can think of the paradox as showing the incompatibility of the welfaristic requirement GPC', the libertarian position

represented by (4.1) and (4.2″), and the behavioural assumption (4.3″) (for convenience, call this Interpretation I). Alternatively, one can view the paradox as showing the incompatibility of the welfaristic requirement of BPC', the libertarian position represented by (4.1) and (4.8), the behavioural assumption (4.9), and REJ' viewed as a normative requirement of social consistency (call this Interpretation II).

(2) Under neither of these two interpretations, the paradox can be regarded as a *direct* tension between Paretianism and libertarian values. This is because, in both the interpretations, specific behavioural assumptions are used to generate the tension under consideration. In addition, Interpretation II needs the requirement of social consistency, REJ'.

(3) The use of (4.3″) in Interpretation I is rather problematic, since the interpretation cannot distinguish between: (i) those cases where, given a rights-structure satisfying (4.1) and (4.2″), the behavioural assumption (4.3″) cannot possibly be satisfied (assuming that, for every game, there must be some outcome); and (ii) those cases where, given a rights structure satisfying (4.1) and (4.2″), it is possible for the behavioural assumption (4.3″) to be satisfied, but GPC' is not satisfied if (4.3″) holds.

(4) While REJ', which figures in Interpretation II, has some normative appeal, even in the absence of any Paretian conditions, REJ' may clash with our intuition about individual rights to liberty in private affairs, if (4.9) holds.

In interpreting the liberal paradox in the framework of the game form approach to individual rights, I have tried to be as faithful as possible to the formal structure of Sen's theorem. However, if one does not adhere rigidly to this formal structure, then it is possible to state and prove more plausible versions of the liberal paradox in the game form framework, which, though departing from Sen's formal structure, are faithful to the intuition of his result. For such analysis, the reader may refer to Campbell (1994) and Deb, Pattanaik and Razzolini (1995).

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