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Welfare, Rights, and Social Choice Procedure: A Perspective*

Abstract: Sen's "The Impossibility of a Paretian Liberal" was meant to crystallize his fundamental criticism against the welfaristic basis of welfare economics in general, and social choice theory in particular. This paper vindicates Sen's criticism, arguing that its logical relevance is not lost in light of recent criticisms against his method of articulating individual rights in terms of a person's decisive power in social choice. We show that some recent proposals that Sen's articulation failed to capture a strong libertarian tradition of free contract and that an appropriate formulation of this tradition wipes off the Sen impossibility cannot be sustained. We then show that the game form articulation of rights casts serious doubts on the Sen articulation of liberty, but the Sen impossibility strenuously comes back in the context of realizing conferred game form rights as well as in the context of initial conferment of game form rights.

1. Introduction

It is slightly ironic that the *Bergson-Samuelson social welfare function* and the *Arrow social welfare function*, which have so much to contrast with each other in many important respects, have a basic feature in common.¹ Despite

* For many helpful discussions on this and related issues over many years, I am most grateful to Professors Kenneth J. Arrow, Peter J. Hammond, Prasanta K. Pattanaik and Amartya K. Sen.

¹ At the risk of reminding readers of what is obvious to them, note that the Bergson-Samuelson social welfare function is "a function of all the economic magnitudes of a system which is supposed to characterize some ethical belief Any possible opinion is admissible We only require that the belief be such as to admit of an unequivocal answer as to whether one configuration of the economic system is 'better' or 'worse' than any other or 'indifferent', and that these relationships are transitive The function need only be ordinally defined A more extreme assumption . . . states that individuals' preferences are to 'count'. If any movement leaves an individual on the same indifference curve, then the social welfare function is unchanged, and similarly for an increase or decrease." (Samuelson 1947, 221–228) In contrast, "[b]y [the Arrow] social welfare function will be meant a process or rule which, for each set of individual orderings R_1, \dots, R_n for alternative social states (one ordering for each individual), states a corresponding social ordering of alternative

the fact that it is the Arrow impossibility theorem and nothing else that poses a devastating criticism against the possibility of the *democratic* Bergson-Samuelson social welfare function, both concepts hinge on the informational basis which is *welfaristic* in nature.²

In the case of the Bergson-Samuelson social welfare function, this fact is quite explicit. For each profile $\mathbf{u} = (u_1, u_2, \dots, u_n)$ of ordinal individual utilities, where u_i ($i = 1, 2, \dots, n$) denotes person i 's ordinal utility and n denotes the number of persons in the society, the Bergson-Samuelson social welfare function f maps \mathbf{u} into an ordinal index of social welfare: $u = f(\mathbf{u})$.³ Thus, social welfare judgements in accordance with the Bergson-Samuelson social welfare function depend on the information of individual utilities and nothing else. In the case of the Arrow social welfare function F , which maps each profile $\mathbf{R} = (R_1, R_2, \dots, R_n)$ of individual preference orderings over the set X of all conceivable social states, where R_i ($i = 1, 2, \dots, n$) denotes person i 's individual preference ordering, into a social preference ordering $R = F(\mathbf{R})$, this fact is less conspicuous.⁴ However, it is known that the Arrow social welfare function satisfies the following property with strong welfaristic flavour.⁵

Strong Neutrality: For any pairs $\{x, y\}$ and $\{a, b\}$ of social states and for any profiles $\mathbf{R}^1 = (R_1^1, R_2^1, \dots, R_n^1)$ and $\mathbf{R}^2 = (R_1^2, R_2^2, \dots, R_n^2)$ of individual preference orderings, if xR_i^1y holds if and only if aR_i^2b holds for all $i = 1, 2, \dots, n$, then xR^1y holds if and only if aR^2b holds, where $R^1 = F(\mathbf{R}^1)$ and $R^2 = F(\mathbf{R}^2)$.

Thus, social welfare judgements in accordance with the Arrow social welfare function depend on the information of relative positions of social states in the individual preference orderings and all other characteristic features of

social states, R. ... [The Arrow] social welfare function ... is a method of choosing which social welfare function of the Bergson type will be applicable Since we are trying to describe social welfare and not some sort of illfare, we must assume that the social welfare function is such that the social ordering responds positively to alterations in individual values, or at least not negatively. Hence, if one alternative social state rises or remains still in the ordering of every individual without any other change in those orderings, we expect that it rises, or at least does not fall, in the social ordering" (Arrow 1951, 23–25)

² According to Sen 1979b, 464, "[w]elfarism implies that any two states of affairs that are identical in terms of individual utility characteristics must be judged to be equally good no matter how different they are in nonutility respects, and also that any state that has more utility for someone and no less utility for anyone in comparison with another state is a better state than the other." The latter property, which is called the *Pareto Principle*, is also shared by the Bergson-Samuelson social welfare function and the Arrow social welfare function.

³ See Samuelson 1947, 228.

⁴ An *ordering* R on a set X is a binary relation defined over X satisfying: (a) [*Completeness*]: For any x, y in X , either xRy or yRx holds; and (b) [*Transitivity*]: For any x, y and z in X , xRy and yRz imply xRz . A *preference ordering* R on X is defined to mean that xRy holds if and only if x is at least as preferable as y . When x is strictly preferred to y , viz. when xRy holds but yRx does not hold, we write $xP(R)y$.

⁵ See Sen 1977b; 1979a for the formal proof of this important fact.

social states are deemed completely irrelevant. It is against this common welfaristic feature that underlies traditional welfare economics and social choice theory that Sen's "Impossibility of a Paretian Liberal" was meant to cast a serious doubt. Whatever else we may say for or against Sen's impossibility theorem, it is in this arena that the value of his contribution should be tested in the final analysis. *Hic Rhodes, hic salta.*

The structure of the paper is as follows. In Section 2, Sen's original formulation of the concept of individual liberty and his impossibility theorem are briefly recapitulated. Section 3 examines Sen's formulation of individual liberty in the light of several criticisms raised against it. In Section 4, we identify three crucial problems that should be squarely examined by any theoretical approach to the concept of individual liberty. Section 5 is devoted to the evaluation of Sen's criticism against the welfaristic foundation of normative economics. Section 6 concludes.

2. Sen's Concept of Individual Liberty and the Impossibility of a Paretian Liberal

Sen's concept of individual liberty is phrased in the context of social choice framework which is slightly more general than that of Arrow (1951). Let X and $N = \{1, 2, \dots, n\}$, where n is a finite integer which is no less than 2, be the set of all conceivable social states and the set of all persons in the society. Σ denotes a family of non-empty subsets of X . Each element of Σ is meant to denote an *opportunity set*, which the society faces under suitably specified conditions. It is assumed that there exists no restriction on how the individual evaluates social states from his/her idiosyncratic point of view. Thus, each and every person can have whatever preference ordering over X he/she cares to express. Given any profile $\mathbf{R} = (R_1, R_2, \dots, R_n)$ of individual preference orderings, and given any opportunity set S in Σ , the society must choose something from S , paying proper attention to the distribution of persons' wishes which is summarized by \mathbf{R} . Let $C(S, \mathbf{R})$ be the non-empty subset of S consisting of all social states which the society chooses from S when \mathbf{R} summarizes peoples' wishes. $C(S, \mathbf{R})$ is to be called the *social choice set* for (S, \mathbf{R}) . A function C which is defined on the Cartesian product of Σ and Ω , where Ω stands for the set of all logically conceivable profiles, and maps each (S, \mathbf{R}) into $C(S, \mathbf{R})$ will be called the *collective choice rule*.

Let us say that a group D of persons is decisive over a pair $\{x, y\}$ of social states if and only if D can secure that y (resp. x) does not belong to $C(S, \mathbf{R})$ as long as x (resp. y) is available in S by expressing unanimous preference within D for x (resp. y) against y (resp. x). If it so happens that a singleton set $\{i\}$ is

decisive over $\{x, y\}$ for some person i in N , we say that the person i is decisive over $\{x, y\}$. We are now ready to state the following:⁶

Sen's Minimal Liberty: There are at least two persons such that for each of them there is at least one pair of social states over which he/she is decisive.

The intended meaning of this condition is illustrated by Sen as follows:

“Given other things in the society, if you prefer to have pink walls rather than white, then [the] society should permit you to have this, even if a majority of the community would like to see your walls white. Similarly, whether you should sleep on your back or on your belly is a matter in which the society should permit you absolute freedom, even if a majority of the community is nose enough to feel that you must sleep on your back.” (1970b, 152)

Note that, to be concordant with this intuitive justification, the pair of social states which are mentioned in Sen's condition should be such that they differ only in the mentioned person's personal matter.

To make this crucial point explicit, let X_0 denote the set of all *impersonal* features of the society, and let X_i , where i is any element in N , denote the set of all *personal* features of person i . Then, X is the Cartesian product of X_0, X_1, \dots, X_n , and each and every social state x is represented by an $(n+1)$ -tuple of feature alternatives: $x = (x_0, x_1, \dots, x_n)$, where x_0 is taken from X_0 and x_i for each i in N is taken from X_i . For convenience, let x_{-i} for each i in N be defined by $x_{-i} = (x_0, x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$ and let x be denoted alternatively as $x = (x_i; x_{-i})$. Let j and k be two persons mentioned in Sen's condition, and let $\{x^i, y^i\}$ ($i = j, k$) be the pair of social states over which person i is decisive. In order for Sen's formulation to be consistent with his intuitive concept of individual liberty, we must have $x_{-i}^j = y_{-i}^j$ for $i = j, k$, so that x^i and y^i differ only in person i 's personal feature.

Turning to the other requirement on collective choice rule, let us now introduce a widely known condition which is welfaristic in nature:⁷

Pareto Principle: If every person in the society prefers any social state x to another social state y , then y should never be socially chosen from any opportunity set S which contains x .

Since the Pareto Principle has seldom been seriously challenged as a reasonable requirement on social welfare judgements, there is no wonder that Sen's impossibility theorem to the effect that *there exists no collective choice*

⁶ There are many versions of Sen's condition of minimal liberty, depending on how we specify the social choice framework as well as on how we define *social preference*. See, among others, Sen 1976; 1979b; 1983; 1992, and Pattanaik 1994; 1996. The version used in the text is taken from Sen 1970b, 156, footnote 4. Whichever version we may pick from among the many alternatives, the following points basically hold *mutatis mutandis*.

⁷ This version of the Pareto Principle is also taken from Sen 1970b, 156.

rule satisfying Sen's minimal liberty as well as the Pareto principle caused a stir. As Sen (1970b, 157) put it in his first paper on the *impossibility of a Paretian liberal*, "the moral [of this impossibility theorem] is that in a very basic sense liberal values conflict with the Pareto principle. ... [I]f someone does have certain liberal values, then he may have to eschew his adherence to Pareto optimality." A truly devastating criticism against the welfaristic basis of normative economics indeed.

Before proceeding to the critical examination of Sen's condition of minimal liberty, it may be worth examining Gibbard's (1974) extension of this condition. The gist of his extension is that if a person, say i , is warranted by the society's collective choice rule to be decisive over $\{x^i, y^i\}$, where $x^i = (x_i^i; x_{-i}^i)$ and $y^i = (y_i^i; x_{-i}^i)$, it does not make much intuitive sense to deny i 's decisiveness over $\{z^i, w^i\}$, where $z^i = (x_i^i; z_{-i}^i)$ and $w^i = (y_i^i; z_{-i}^i)$. After all, if Ian is empowered to paint his bedroom walls pink rather than white when all other persons paint theirs yellow, why should we not empower him to use pink rather than white when all other persons are using blue instead? Likewise, why should we not empower John, Kevin and Liz to choose the colour of their bedroom walls freely when we empower Ian in this way? Presumably, it was these considerations that led Gibbard (1974) to formulate the following natural extension of Sen's condition:

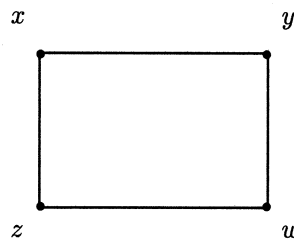
Gibbard's Libertarianism. Each person i in N is decisive over the pair of social states $\{x^i, y^i\}$, where $x^i = (x_i; x_{-i})$ and $y^i = (y_i; x_{-i})$, whatever may be the specification of x_i, y_i , and x_{-i} .

Despite the common intuitive root of Sen's minimal liberty and Gibbard's libertarianism, the logical consequence of Gibbard's libertarianism is even more disturbing than Sen's impossibility of a Paretian liberal. Indeed, it is shown by Gibbard (1974) that *there exists no collective choice rule satisfying Gibbard's libertarianism*. The gist of this result can be illustrated by the situation where $N = \{1, 2\}$, $X_0 = \{x_0\}$ and $X_i = \{a, b\}$ for $i = 1, 2$. Let $x = (x_0, a, a)$, $y = (x_0, a, b)$, $z = (x_0, b, a)$ and $w = (x_0, b, b)$. Suppose that the two persons have the following preference orderings: $xP(R_1)z$, $zP(R_1)w$ and $wP(R_1)y$ for person 1, and $zP(R_2)w$, $wP(R_2)y$ and $yP(R_2)x$ for person 2. Given this profile $\mathbf{R} = (R_1, R_2)$ and an opportunity set $S = \{x, y, z, w\}$, Gibbard's libertarianism dictates that the social choice set $C(S, \mathbf{R})$ cannot but be empty, given the decisiveness of person 1 (resp. person 2) over $\{x, z\}$ and $\{y, w\}$ (resp. $\{x, y\}$ and $\{z, w\}$). Whichever state in S the society chooses, it cannot but violate either person 1's or person 2's decisiveness. The moral is that Sen's concept of liberty in the form generalized by Gibbard generates a system of individual claim rights to collective choice rule, which is self-contradictory.

The Gibbard impossibility theorem leads us to an interesting further question. Under what conditions can we assure the existence of a collective choice

rule which materializes a system of individual claim rights generated by Sen's requirement of individual liberty? A complete answer to this question may be found in Suzumura (1978; 1979; 1983, ch. 7), but the essence of the answer is simple, which can be intuitively illustrated in terms of Figure 1. Note that person 1 is decisive over $\{x, z\}$ and $\{y, w\}$ and person 2 is decisive over $\{x, y\}$ and $\{z, w\}$. Thus, we can start from any state in S , say x , and follow a path along the edges of the rectangle in Figure 1, say from x to y to w to z , and come back to x again. Along this loop, each and every edge consists of a pair of social states over which either person 1 or person 2 has decisiveness. It is the existence of such a critical loop that underlies the Gibbard impossibility theorem. Excluding the occurrence of such a critical loop is necessary and sufficient for the existence of a collective choice rule which materializes a system of individual claim rights generated by Sen's requirement of individual liberty.

Figure 1: Gibbard's Impossibility Theorem



Note: person 1 has decisiveness over $\{x, z\}$ and $\{y, w\}$, while person 2 has decisiveness over $\{z, w\}$ and $\{x, y\}$.

3. Sen's Formulation of Individual Liberty: Critical Examination

Given the basic nature of Sen's criticism, it is all too natural that the impossibility of a Paretian liberal has been under careful scrutiny along several lines.⁸ Given our present purpose, we have only to focus on the way in which Sen crystallized his intuition on individual liberty in terms of the analytical framework of social choice theory.

The first misgivings, which are frequently expressed in the literature, criticize Sen's formulation of individual liberty as having failed to consider "a strong libertarian tradition of free contract", according to which "a person's rights are for his to use or to bargain away as he finds fit" (Gibbard 1974, 397). This viewpoint was most conspicuously formulated by Harel and Nitzan

⁸ For surveys of some of these works, see Sen 1976; 1983; 1992, Suzumura 1983, ch. 7, and Wriglesworth 1985.

(1987). It is through the careful examination of their proposal that we can pinpoint the crucial problem underlying this escape route from Sen's impossibility theorem.⁹

The gist of the Harel-Nitzan proposal can be crystallized in terms of a simple example due to Sen (1970b). There is a single copy of *Lady Chatterley's Lover*. Everything else being the same, there are three social states: Mr. *P* (the prude) reading it (r_P), Mr. *L* (the lascivious) reading it (r_L), and no one reading it (r_0). Mr. *P* ranks them in the descending order of r_0, r_P, r_L , whereas Mr. *L* ranks them in the descending order of r_P, r_L, r_0 . Since to read a book or not is ordinarily construed as a person's private matter and no other person's business, Sen endows Mr. *P* (resp. Mr. *L*) with decisiveness over $\{r_P, r_0\}$ (resp. $\{r_L, r_0\}$).¹⁰ Given this system of claim rights based on the decisiveness of persons, and given the profile $\mathbf{R} = (R_P, R_L)$ of individual preference orderings we have specified, the social choice set $C(\{r_P, r_L, r_0\}, \mathbf{R})$ cannot but be empty, vindicating Sen's impossibility theorem. In this situation, Harel and Nitzan call our attention to the fact that Mr. *P* (resp. Mr. *L*) has *ordinally stronger preference* for r_0 against r_L than that for r_0 against r_P (resp. for r_P against r_0 than that for r_L against r_0).¹¹ Thus, so the Harel-Nitzan argument goes, Mr. *P* has incentive to exchange his claim right based on his decisiveness over $\{r_0, r_P\}$ with the claim right of Mr. *L* based on his decisiveness over $\{r_L, r_0\}$. Mr. *L* is similarly motivated. If this mutually beneficial exchange of claim rights are in fact realized between Mr. *P* and Mr. *L*, bringing Mr. *P* (resp. Mr. *L*) to be decisive over $\{r_L, r_0\}$ (resp. $\{r_0, r_P\}$), then the impossibility result identified by Sen evaporates. Indeed, the social choice after the realization of voluntary exchange of claim rights will be $\{r_P\}$.

Note, however, that this 'resolution' of the impossibility of a Paretian liberal has very little to commend itself to a person with liberal belief in the ordinary sense of the word. Indeed, to enable Mr. *A* to choose whether Mr. *B* should or should not read a book, not in view of Mr. *B*'s own preferences but in view of Mr. *A*'s preferences, is not liberalism but paternalism, and a liberal may well regard paternalism as the worst form of despotism imaginable.

The problem with the Harel-Nitzan scheme does not end there. Consider the situation where $N = \{1, 2\}$, $X = \{v, w, x, y, z\}$ and person 1 (resp. person 2) is decisive over $\{x, y\}$ and $\{v, z\}$ (resp. $\{v, x\}$ and $\{w, z\}$). As is clear from the LHS of Figure 2, there is no critical loop in the system of claim rights

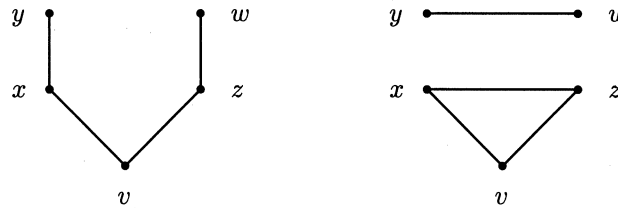
⁹ The following analysis is based on Suzumura 1991. See also Breyer 1990.

¹⁰ Note that there exists no critical loop in the distribution of decisiveness in this example, so that the Gibbard impossibility theorem does not have any bite in this context.

¹¹ When a preference ordering R is such that $xP(R)y$, $yP(R)z$ and $xP(R)z$ hold, we say that the preference for x against z is *ordinally stronger* than that for x against y . Likewise, the preference for x against z is *ordinally stronger* than that for y against z . It was Blau 1975 who introduced this concept into social choice theory, but the origin of the concept goes back at least as far as to Luce and Raiffa 1957.

generated by this distribution of decisive power. Suppose that the profile of individual preference orderings are as follows: $xP(R_1)v$, $vP(R_1)w$, $wP(R_1)y$, $yP(R_1)z$, $vP(R_2)y$, $yP(R_2)z$, $zP(R_2)x$, and $xP(R_2)w$. It is clear that person 1 has ordinally stronger preference for x against z than that for x against y . Likewise, person 2 has ordinally stronger preference for y against w than that for z against w . In the situation like this, Harel and Nitzan allow the two persons to realize the mutually beneficial exchange of *social states* y and z to create a new pair $\{x, z\}$ for 1 and $\{w, y\}$ for 2, over which they are decisive. This is obviously bizarre. To exchange a pair of social states, over which a person has a claim right, with another pair of social states, over which the exchange partner has a claim right, has a clear meaning, but to exchange social states between persons so as to concoct new decisive pairs of social states does not make any sense at all. Worse still, if this bizarre exchange is somehow enforced, the resulting assignment of claim rights has a critical loop, even though such a loop did not exist before the exchange. See the RHS of Figure 2.

Figure 2: Logical Difficulty of Harel-Nitzan Libertarian Right



Note: In LHS, 1 (resp. 2) has decisiveness over $\{x, y\}$ and $\{v, z\}$ (resp. $\{v, x\}$ and $\{w, z\}$), while in RHS, 1 (resp. 2) has decisiveness over $\{x, z\}$ and $\{v, z\}$ (resp. $\{w, y\}$ and $\{v, x\}$).

We cannot but conclude that the criticism against Sen's formulation of individual liberty along this line has serious problems of its own, and does not succeed in presenting a meaningful alternative concept of individual liberty, let alone a 'resolution' of the impossibility of a Paretian liberal.

There is another string of critics who also find Sen's articulation of individual liberty in terms of decisiveness rather at odds with what an ordinary liberal would claim. To bring this point home, recollect Sen's motivation for his minimal liberty condition to the following effect: "If you prefer to have pink walls rather than white, then [the] society should permit you to have this" and also that "whether you should sleep on your back or on your belly is a matter in which the society should permit you absolute freedom". Not many people with liberal belief would have anything to say against Sen's intuitive motivation. However, the actual formulation of this intuition in terms

of the relevant person's decisiveness in social choice may make such a person raise an eyebrow. He/she may well ask: Why don't we simply leave the matter of choosing the colour of one's bedroom walls, or choosing one's sleeping posture, to the relevant person's warranted *individual* choice, rather than articulating such a right through his/her decisiveness in *social* choice?

It was in this vein that Nozick made the following famous remark on Sen's impossibility of a Paretian liberal:

"A more appropriate view of individual rights is as follows. Individual rights are co-possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. ... If I have a right to choose to live in New York or in Massachusetts, and I choose Massachusetts, then alternatives involving my living in New York are not appropriate objects to be entered in a social ordering."
(Nozick 1974, 166)

Capitalizing on, and generalizing Nozick's observation, Sugden (1978; 1985a; 1985b) and Gaertner, Pattanaik and Suzumura (1992) have developed an alternative approach to individual libertarian rights, which came to be known as the *game form approach* to individual rights.¹² This approach articulates individual libertarian rights as (i) the complete freedom of each player to choose any admissible strategy, and (ii) the obligation of each player not to choose an inadmissible strategy for himself/herself, and not to prevent anyone from choosing an admissible strategy.

In the case of Nozick's counterargument against Sen, for example, the game form that captures Nozick's right to choose to live in New York or Massachusetts can be formulated neatly as follows: Nozick's set of admissible strategies, say S_{Nozick} , should contain 'to live in New York' and 'to live in Massachusetts', and the set of admissible strategies for all other persons should not contain such strategies as 'to harass Nozick if he chooses to live in Massachusetts', 'to force Nozick to live in New York at gunpoint', and so forth, and the outcome function g should be such that $g(s)$ is a social state in which Nozick lives in Massachusetts (resp. he lives in New York) if s_{Nozick}

¹² A *game form* is a specification of a set N of *players*, a set S_i of *admissible strategies* for each player i in N , a set A of *feasible outcomes*, and an *outcome function* g which maps each strategy profile $s = (s_1, s_2, \dots, s_n)$, where s_i is in S_i for each i in N , into a social outcome $g(s)$ in A . Given a game form $G = (N, \{S_i\}, g)$, if a profile $R = (R_1, R_2, \dots, R_n)$ of preference orderings of the players is specified, we have a *game* (G, R) . Gärdenfors 1981 developed a related but distinct game theoretic approach to individual liberty. See also Bernholz 1974, Deb 1994, Gibbard 1982, Hammond 1995; 1996, Pattanaik 1994; 1996, Sen 1981; 1992, and Suzumura 1990 for more detailed account of the alternative approaches to individual liberty. The following exposition of the game form approach is based on Pattanaik and Suzumura 1994; 1996.

(viz. Nozick's component of s) is 'to live in Massachusetts' (resp. 'to live in New York').¹³

Two remarks are in order at this juncture. First, unlike the first alternative approach based on the voluntary exchange of libertarian rights, which not only accused Sen's approach of being out of line with traditional liberal values, but also asserted that the impossibility of a Paretian liberal could be resolved by appropriately reformulating what a liberal should claim, the game form approach does not claim to be a resolvent of Sen's impossibility theorem. Quite to the contrary, it was conjectured that the Sen impossibility problem "persists under virtually every plausible concept of individual right..." (Gaertner/Pattanaik/Suzumura 1992, 161). We will have more to say on this point in the next section.

Second, the game form articulation of individual libertarian rights based on the intuitive concept of freedom of choice is not just an alternative approach to Sen's classical articulation of individual liberty. It is also meant to cast serious doubt on Sen's approach. To bring this point home, let us examine a modified version of the *Lady Chatterley's Lover* case. Suppose that both Mr. P and Mr. L own a copy of this book. Everything else remaining the same, there exist four social states: (r, r) , (r, n) , (n, r) and (n, n) , where r (resp. n) stands for 'to read it' (resp. 'not to read it'). Suppose further that their preference orderings over $\{(r, r), (r, n), (n, r), (n, n)\}$ are described as follows:¹⁴

$$R_P : (n, n), (r, r), (n, r), (r, n)$$

$$R_L : (n, r), (r, n), (r, r), (n, n).$$

Following the game form approach and the intuitive concept of freedom of choice, let us entrust each and every person to choose either to read this book or not to read it in accordance with his individual preference. However, this is not a straightforward problem of preference optimization. The effect on a person of his choice from the set of options $\{r, n\}$ hinges squarely on what the other person chooses from the same set of options, and no one is within his right to know the other's choice beforehand. In this sense, the problem of choice faced by Mr. P and Mr. L is that of choice under uncertainty. If they follow the maximin principle of choice under uncertainty, the maximin choice of Mr. P (resp. Mr. L) is n (resp. r), thereby generating a social state (n, r) through unhindered exercise of their respective freedom of choice. However, since (r, r) and (n, r) differ only in Mr. P 's reading or not reading

¹³ Lest we should be misunderstood that the game form approach hinges on the supposition that each and every person is empowered to control some aspects of social states *directly*, let us emphasize that no such unwarranted restriction is needed for the workability of this approach. Those who are interested are referred to Gaertner, Pattanaik and Suzumura 1992, Pattanaik 1996, and Suzumura 1990, where many examples are worked out in order to illustrate and substantiate this claim.

¹⁴ Preference orderings are represented horizontally, with the less preferred alternative to the right of the more preferred alternative.

this book, and Mr. P prefers (r, r) to (n, r) , the realization of (n, r) cannot but be regarded that Mr. P 's liberty is violated if we subscribe to Sen's articulation of individual liberty, even though nobody's freedom of choice is violated in this case.

We have thus shown that the game form articulation of individual liberty is a viable alternative to Sen's formulation, and it poses a serious doubt on the compatibility of Sen's approach with our intuition about freedom of choice. In the next section, we will identify three crucial problems in the theory of individual rights within the conceptual framework of game form approach.

4. Articulation, Realization, and Conferment of Rights

In discussing individual libertarian rights, three distinct issues should be addressed. The first issue is the *formal structure* of rights. The second issue is the *realization* of conferred rights. The third issue is the *initial conferment* of rights. In the previous section, we have contrasted Sen's articulation of the formal structure of individual libertarian rights and the game form articulation of rights. In Sen's approach, the issue of the realization of conferred rights boils down to the existence of a collective choice rule which realizes the conferred individual decisiveness in social choice, whereas Sen never addressed himself to the issue of the initial conferment of rights.¹⁵ The rest of this section is devoted to explaining how the game form approach treats the second and third issues.¹⁶

The issue of the realization of conferred rights is treated by the game form approach as follows. Let A be the set of feasible social states. Given a game form $G_A = (N, \{S_i\}, g_A)$ which articulates the conferred individual rights when A prevails, and given a profile $\mathbf{R} = (R_1, R_2, \dots, R_n)$ of individual preference orderings over the set of social states, we have a game (G_A, \mathbf{R}) . Let $T(G_A, \mathbf{R})$ be the set of all social states which the society predicts to appear when the game (G_A, \mathbf{R}) is played.¹⁷ It is clear that the conferred rights G_A will be realized through the play of the game (G_A, \mathbf{R}) and a social state in $T(G_A, \mathbf{R})$ will materialize as a result of the play of this game.

¹⁵ Since Sen's interest was focussed on the basic conflict between non-welfaristic claim of rights and welfaristic claim of Pareto optimality, it was unnecessary for him to provide a full characterization of rights, neither was it necessary for him to develop a theory of the initial conferment of rights.

¹⁶ The following analysis is essentially based on Pattanaik and Suzumura 1994; 1996. Those who are interested in some technical details are referred to these original sources.

¹⁷ If the prevailing concept of equilibrium is given by E and the set $B_E(G_A, \mathbf{R})$ of pure strategy equilibria is non-empty, then it is natural to assume that: $T(G_A, \mathbf{R}) = \{x^* \in A \mid x^* = g_A(s) \text{ for some } s \in B_E(G_A, \mathbf{R})\}$. The case where there exists no pure strategy equilibrium, but a mixed strategy equilibrium does exist, or the case where there exists no equilibrium are discussed in Pattanaik and Suzumura 1996.

The issue of the initial conferment of rights requires us to expand our conceptual framework rather substantially. To ask and answer how and why a game form representing individual rights come to be conferred in the first place, it is not enough that we are informed of the individual preference orderings over the set of social outcomes. To bring this point home, consider the following simple problem.

A father is to divide a cake among three children fairly. Method I is that the father divides this cake into three equal pieces, and tells them to take a piece each or leave it. Method II is that the children are given the opportunity to discuss how this enticing cake should be divided fairly among them, and cut it into three pieces in accordance with the conclusion they arrive at. If they happen to conclude that equal division should be the outcome, and if we are informed only of the outcomes, we cannot but conclude that these two methods are the same. It is clear, however, that this is certainly inappropriate. In the case of method I, three children are not provided with any right to participate in the process through which their dividend is determined, whereas in the case of method II, they are endowed with such a right. To capture this important difference, we must enlarge the description of social state in such a way that, not only the social *outcomes*, but also the *process* or *mechanism* through which such outcomes are brought about, are included.¹⁸

This conceptual expansion can be attained as follows. Let x and y be two (conventionally defined) social states, and let θ and η be two decision-making mechanisms. The ordered pair (x, θ) [resp. (y, η)] denotes an extended social state in which the outcome x (resp. y) is entailed through the decision-making mechanism θ (resp. η). It is assumed that people are prepared to make judgements of the following type: It is better to obtain an outcome x through a mechanism θ than to obtain an outcome y through a mechanism η . In what follows, we focus on the situation where the decision-making mechanism is specified by the *rights-system* \mathbf{G} which specifies a game form G_A for each set of feasible outcomes A . Let $\mathbf{Q} = (Q_1, Q_2, \dots, Q_n)$ be the profile of extended individual preference orderings over the pairs (x, \mathbf{G}^1) , (y, \mathbf{G}^2) , etc. Note in passing that, for any fixed rights-system \mathbf{G} , the profile \mathbf{Q} induces a profile $\mathbf{Q}_{\mathbf{G}} = (Q_{1\mathbf{G}}, Q_{2\mathbf{G}}, \dots, Q_{n\mathbf{G}})$ over the set of conventionally defined social states by $xQ_{i\mathbf{G}}y$ if and only if $(x, \mathbf{G})Q_i(y, \mathbf{G})$ for all x, y in X and all i in N .

Suppose that a feasible set of outcomes A , a rights-system \mathbf{G} , and a profile of extended individual preference orderings \mathbf{Q} are given. We then obtain a game $(G_A, \mathbf{Q}_{\mathbf{G}})$, the play of which will determine a set $T(G_A, \mathbf{Q}_{\mathbf{G}})$ of realizable social states. For the sake of simplicity in exposition, it is assumed in what

¹⁸ See Arrow 1951, ch. 7, sec. 6, and Sen 1995 for further forceful endorsement of this viewpoint.

follows that $T(G_A, Q_G)$ consists only of a single element, say $\tau(G_A, Q_G)$.¹⁹ In this case, a feasible extended social state is given by $(\tau(G_A, Q_G), G)$.

We are now ready to explain how this framework treats the issue of the initial conferment of rights. Let Ψ be the *extended social welfare function* which maps each profile $Q = (Q_1, Q_2, \dots, Q_n)$ of extended individual preference orderings into an extended social welfare ordering: $Q = \Psi(Q)$. Given a set A of feasible social states, the socially optimal conferment of rights is nothing other than the rights-system G^* such that $(\tau(G_A^*, Q_{G^*}), G^*) \Psi(Q)(\tau(G_A, Q_G), G)$ holds for any feasible rights-system G .

Before closing this summary account of the game form approach to individual libertarian rights, two remarks are due. First, unlike Sen's classical articulation of rights, the game form articulation of rights does not assign any role whatsoever to individual preferences. However, in the realization of rights articulated by the rights-system, as well as in the initial conferment of rights, this theory does assign crucial role to the profile of extended individual preference orderings. In the former case, it is the induced preference profile Q_G , together with the set A of feasible social states, which determines the game (G_A, Q_G) to be played as well as the outcome of the play $\tau(G_A, Q_G)$. In the latter case, it is the extended social welfare ordering $Q = \Psi(Q)$ which determines the rights-system to be conferred. Thus, in the full theory of game form approach to rights, there are important niches for individual preference orderings. To recapitulate, although the formal contents of the conferred game form rights are independent of individual preferences, the extended individual preferences play a crucial role in deciding the rights-system to be conferred, as well as in socially realizing the individual freedom of choice thus conferred.

Second, unlike in the context of the Sen-Gibbard rights, the game form approach to rights does not have any counterpart of the Gibbard impossibility theorem. In other words, the problem of internal inconsistency of rights never surfaces in the game form approach. To the extent that the initial conferment of rights is performed in accordance with the scenario of the game form approach, the conferred rights will be realized through the actual play of the game, thereby excluding any possibility of internal inconsistency of rights.

5. Sen's Criticism Against Welfarism: An Evaluation

Back, then, to the central focus of this paper. What does the game form approach clarify about the impossibility of a Paretian liberal? Does it fortify, or qualify, or even nullify Sen's criticism against welfarism which is based on the basic conflict between the welfaristic Pareto principle and the non-

¹⁹ See Pattanaik and Suzumura 1996 for a fuller exposition without this simplifying assumption.

welfaristic claim of individual liberty? In what follows, we will contend that the main thrust of Sen's criticism against welfarism remains intact even if Sen's articulation of individual liberty is rejected and replaced by the game form articulation.

To begin with, consider yet another variant of Sen's *Lady Chatterley's Lover* case. As in the first variant used in Section 3 to crystallize a conceptual difficulty of Sen's approach, suppose that both Mr. P and Mr. L have a copy of *Lady Chatterley's Lover*, and their preference orderings over the set of feasible social states $\{(r, r), (r, n), (n, r), (n, r)\}$ are given by:

$$\begin{aligned} R_P &: (n, n), (r, n), (n, r), (r, r) \\ R_L &: (r, r), (r, n), (n, r), (n, n). \end{aligned}$$

As in the first variant, the issue of individual liberty contained in this situation may be captured neatly by the game form $G = (N, \{\{n, r\}, \{n, r\}\}, g)$, where $N = \{P, L\}$ and the outcome function g is such that $g(s_P, s_L) = (s_P, s_L)$, where s_P and s_L are taken from $\{n, r\}$. Unlike in the first variant, however, the preference profile $\mathbf{R} = (R_P, R_L)$ that defines a game (G, \mathbf{R}) has a *dominant strategy equilibrium* (n, r) , which is Pareto dominated by (r, n) . Thus, the voluntary exercise of freedom of choice yields a social state which is Pareto dominated by another feasible social state. This is the first instance in which Sen's impossibility of a Paretian liberal recurs in the context of realizing conferred game form rights.

As a matter of fact, Sen's impossibility recurs also in the context of initial conferment of game form rights. To show this possibility unambiguously, consider a situation where $N = \{C, D\}$ ($C =$ 'consequentialist'; $D =$ 'deontologist'). There are two issues to be decided on. The first issue is the religion, and there are two options: $b =$ 'Buddhism' and $c =$ 'Christianity'. The second issue is whether or not a book is to be read, and there are two options: $r =$ 'to read it' and $n =$ 'not to read it'. Thus, the set A of physically possible social states consists of 16 alternatives. A typical element of A is denoted by $(c, n; b, r)$, which is a state where Mr. C believes in Christianity and does not read the book, and Mr. D believes in Buddhism and reads the book. There are two feasible rights-systems: $G^1 = \{G^1\}$ and $G^2 = \{G^2\}$.²⁰ The game form $G^1 = (N, \{S_i^1\}, g^1)$, where S_i^1 is the Cartesian product of $\{b, c\}$ and $\{r, n\}$ for $i = C, D$ and $g^1(s) = s$ for all $s = (s_1, s_2)$ such that s_i is in S_i^1 for $i = C, D$, is the one where the two persons are empowered to choose their religion as well as reading or not reading the book freely. In contrast, the game form $G^2 = (N, \{S_i^2\}, g^2)$, where $S_i^2 = \{r, n\}$ for $i = C, D$ and $g^2(s) = s$ for all $s = (s_1, s_2)$ such that s_i is in S_i^2 for $i = C, D$, is the one where the two persons are only allowed to choose reading or not reading the book freely, the matter of choosing common religion being decided by the

²⁰ Throughout this example, the feasible set A is fixed, which is why G^1 as well as G^2 consists of only one game form each.

society. If the social choice of common religion is t in $\{b, c\}$ and the strategy pair s is chosen, then the social state will be given by $(t, s_1; t, s_2)$.

Let $Q = (Q_C, Q_D)$ be the profile of extended individual preference orderings. Mr. C is a die hard consequentialist who cares only about the outcomes of social interactions and nothing else. Thus, for all social state x in A , $(x, G^1)I(Q_C)(x, G^2)$ holds true, where $I(Q_C)$ is the indifference relation generated by Q_C . For each pair (u, v) , where u (resp. v) refers to Mr. C 's (resp. Mr. D 's) religion, and for each $G = G^1$ and G^2 , let $Q_{CG}(u, v)$ be defined by:

$$Q_{CG}(u, v) : (u, r; v, r), (u, n; v, r), (u, r; v, n), (u, n; v, n),$$

which, in turn, is used to define Q_{CG} by:

$$Q_{CG} : Q_{CG}(b, c), Q_{CG}(b, b), Q_{CG}(c, b), Q_{CG}(c, c).$$

Mr. D is a deontologist whose belief in the procedural justice in allowing people to choose their religion has such predominant importance that, for all x, y in A , he holds that $(x, G^1)P(Q_D)(y, G^2)$. For each pair (u, v) of religions of Mr. C and Mr. D and for each $G = G^1$ and G^2 , we define $Q_{DG}(u, v)$ by:

$$Q_{DG}(u, v) : (u, n; v, n), (u, n; v, r), (u, r; v, n), (u, r; v, r),$$

which, in turn, is used to define Q_{DG} by:

$$Q_{DG} : Q_{DG}(c, c), Q_{DG}(b, c), Q_{DG}(c, b), Q_{DG}(b, b).$$

Let us examine the game (G^1, Q_{G^1}) . It is easy, if tedious, to check that (b, r) is the dominant strategy for Mr. C , and (c, n) is the dominant strategy for Mr. D . Thus, $(b, r; c, n)$ in A is the dominant strategy equilibrium in the game (G^1, Q_{G^1}) . In the situation where there exists a dominant strategy equilibrium, it is very natural to assume that $\tau(G^1, Q_{G^1}) = ((b, r; c, n), G^1)$. Turning to the game (G^2, Q_{G^2}) , it is again easy to confirm that r (resp. n) is the dominant strategy for Mr. C (resp. Mr. D) irrespective of whether the social choice of religion turns out to be b or c . Thus, $\tau(G^2, Q_{G^2}) = ((b, r; b, n), G^2)$ or $((c, r; c, n), G^2)$ depending on the social choice of b or c . Recollect that Mr. D holds a lexicographic preference for (x, G^1) against (y, G^2) whatever may be x and y . Thus, he must surely prefer $\tau(G^1, Q_{G^1})$ to $\tau(G^2, Q_{G^2})$. Mr. C being a consequentialist, he is indifferent between $\tau(G^1, Q_{G^1}) = ((b, r; c, n), G^1)$ and $((b, r; c, n), G^2)$ and he prefers $((b, r; c, n), G^2)$ to $((b, r; b, n), G^2)$ as well as to $((c, r; c, n), G^2)$. By transitivity of Q_C , Mr. C must then prefer $\tau(G^1, Q_{G^1})$ to $\tau(G^2, Q_{G^2})$. Thus, as long as the extended social welfare function Ψ satisfies the Pareto principle, G^1 must be the rights-system to be conferred. However, if G^1 is conferred and the game (G^1, Q_{G^1}) is played, $(b, r; c, n)$ will be the social outcome, which is Pareto dominated by another feasible social state $(b, n; c, r)$.

We have thus shown that Sen's Pareto libertarian paradox recurs not only in the context of realizing game form rights, but also in the context of initial conferment of game form rights. It is in this sense that we contend that Sen's

criticism against welfarism survives without losing an iota of its importance even if his articulation of libertarian rights has to be replaced by the allegedly more proper game form articulation. We close this section by re-stating our conviction that the Sen impossibility problem “persists under virtually every plausible concept of individual rights”.

6. Concluding Remarks

To argue for the *logical* relevance of Sen’s criticism against welfarism is one thing, and to argue for its *empirical* relevance in the actual context where welfare economics is set in motion is quite another. In this paper, we have confirmed that Sen’s impossibility of a Paretian liberal does not lose its logical relevance even in the light of the many criticisms recently raised against Sen’s method of articulating individual liberty in terms of a person’s decisive power in social choice. How about the empirical relevance? Is Sen’s impossibility just a theoretical curiosity which is amusing as a logical exercise in the classroom, yet can be safely neglected once we turn our attention to the pressing economic problems where the real bite of welfare economics is seriously tested? Quite to the contrary, it seems to us that there are many real situations where serious conflict occurs between the claim of individual rights and the desire for social efficiency.

Suffice it to visualize a local city where small and traditional retailers are engaging in hand-to-mouth business. From the viewpoint of improving social efficiency in retailing service in this city, it makes sense to allow a few large-scale organized retailers to enter this city. If we do so, however, those small retailers who have been doing business in this city over many years will almost surely be unable to cope with the large-scale retailers and expelled from retailing business. Should we pursue the improvement in social efficiency at the cost of depriving small retailers of their ‘rights’ of doing business? Or, should we respect these ‘rights’ at the cost of missing an opportunity to improve social efficiency in retailing business? This is a typical and realistic situation where policy makers are confronted with the conflict between rights and efficiency.

To the extent that welfare economics claims to serve as the theoretical foundations of economic policy, there is no way of avoiding such conflict between two basic values—the welfaristic value of social efficiency, on the one hand, and the non-welfaristic claim of individual rights, on the other. Although Sen (1970b) posed this serious problem in terms of a deceptively simple parable, the problem he thereby posed is neither simple nor unrealistic.

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