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Social Support among Heterogeneous Partners*

Abstract: This paper derives hypotheses on how dyadic social support is affected by heterogeneity of the actors. We distinguish heterogeneity with respect to three parameters. First, the likelihood of needing support; second, the benefits from support relative to the costs for providing support; and, third, time preferences. The hypotheses are based on a game theoretic analysis of an iterated Support Game. We predict that, given homogeneity in two of these parameters, the prospect for mutual support is optimal if actors are homogeneous with respect to the third parameter as well. Second, under heterogeneity with respect to two of the parameters, support is most likely if there is a specific heterogeneous distribution with respect to the other parameter that 'compensates' for the original heterogeneity. Third, under weak conditions, the overall optimal condition for mutual support is full homogeneity of the actors.

1. Introduction

This paper seeks to improve the understanding of cooperation in *asymmetric* social dilemmas. We analyze the conditions under which asymmetry with respect to individual properties hampers or facilitates cooperation. The theoretical and experimental literature on social dilemmas predominantly analyzes cooperation in the *symmetric* Prisoner's Dilemmas and some related social dilemma games. A Prisoner's Dilemma is called symmetric if the payoffs of the actors are the same (up to an increasing affine transformation). Asymmetry studied so far in the literature mainly concerns asymmetry in information, i.e., differences between the actors in the information about each other's payoffs, the rules of the games, etc. Asymmetry in the sense of 'actor heterogeneity' with respect to other aspects of interactions has received much less attention. Actor heterogeneity refers to differences in tastes, interests, resources, etc. between the actors involved. This paper focuses on the consequences of actor heterogeneity on cooperation in games that are a variant of the Prisoner's Dilemma. Asymmetries that do not directly reflect differences in properties of actors, such as differences in positions in social networks or differential institutional treatment of actors will not be addressed in this paper.

We argue that actor heterogeneity likely has different effects on individual behavior and on collective outcomes in different types of social dilemma situations. Our distinction resembles the 'production function' analysis of collective action (Oliver et al. 1985; Heckathorn 1992). In the first type, exemplified by

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the *Prisoner's Dilemma*, Pareto optimal outcomes ('cooperation') require that *each* actor contributes (Rapoport 1974; Dawes 1980), while *all* actors have incentives to deviate. If actors indeed deviate, the resulting outcome is suboptimal (Fudenberg/Tirole 1992; Kreps 1990). A common example is environmental protection. Since time and attention are scarce recourses, everyone has incentives not to separate trash into paper, plastics, metal, glass etc., although everyone may agree that such separation is important to protect the environment. Everyone welcomes that other people separate their trash, but may abstain from such costly behavior him or herself ('free riding'). If everyone does so, however, the resulting environment will be suboptimal. In the second class of problems, exemplified by the *Volunteer's Dilemma*, the production of the collective good requires contributions by a *sufficient number* of actors. An example is the rescue of a child who has wandered too far into the sea. Bystanders have the same aim - the child should be rescued. However, it is not necessary that all bystanders jump into the water to rescue the child. It is sufficient if one actor jumps into the water and rescues the child. This would be a Pareto optimal outcome (Diekmann 1993; Weesie 1993). If more than one actor would jump into the water, the extra efforts are wasted, and the outcome would be Pareto suboptimal. Olson and Zeckhauser (1966) illustrated this effect with contributions to the NATO alliance during the 60's.

In a dilemma situation of the Prisoner's Dilemma type, the problem is the inducement of the actor(s) with the *least* interest in the production of the collective good to make a contribution. In terms of a Prisoner's Dilemma, the problem is the inducement of the actor(s) with the highest incentive to cheat. We want to indicate an analogy with the strength of a chain in terms of the strength of the links: To make a chain stronger, the *weakest link* has to be strengthened. With unlimited material, we can make all links stronger. With scarce resources, we can make the chain stronger by shifting material from the stronger links to the weakest link. In a chain of maximal strength, all links are equally strong, i.e., homogeneity is optimal. To the extent that this analogy is valid, we would expect that actor homogeneity offers optimal prospects for cooperation in a social dilemma of a Prisoner's Dilemma type.

The effect of heterogeneity on efficiency is different in the second class of social dilemmas, exemplified by the Volunteer's Dilemma. In this class of social dilemmas, an outcome, in which all actors contribute, is suboptimal. The production of the collective good requires the contribution by one actor only. Various theoretical analyses suggest that the actor, who gains most from the collective good relative to the costs of producing it, is the likely volunteer. Therefore, the production of the collective good depends on the *strongest link*. Heterogeneity among the actors likely makes the strongest link stronger, and so facilitates cooperation. Different reasons have been suggested for the strong impact of the strongest link. Diekmann (1993) derives this prediction using the theory of risk dominance, one of the prominent equilibrium selection theories of game theory. Weesie (1993) derives a similar prediction, using subgame perfection only, if actors are able to wait and see whether other actors made a contribution. A related argument can be found in mobilization and threshold theories of collective action. Olson (1965) argued that the actors who are more interested in the common good will provide the collective good, regardless of the actions of the less interested persons of the group. The 'weak' can thus exploit the 'strong' because everyone knows that the 'strong' will produce the collective good anyway (Olson 1965, 29ff.). Oliver et al. (1993) give a similar argument in their analysis of the critical mass of heterogeneous actors and collective action. They argued that heterogeneity of interests or resources facilitates collective action because it increases the likelihood that a 'critical mass' of highly motivated contributors will emerge to initiate the collective action (Oliver et al. 1985; similar argument in Hardin 1982). Coleman (1990) argues that organizing collective action is more likely in a group that is heterogeneous with respect to normative constraints than in a group with homogeneous normative constraints.

The modeling in this area has become increasingly sophisticated, but the main underlying argument remains the same (see e.g. Heckathorn 1992; Granovetter 1978; 1980; Rapoport 1988; Rapoport et al. 1989). What all of the 'critical mass' theories or analyses of the Volunteer's Dilemma like collective good problems have in common is that the argument of the 'weakest link' is replaced by a 'strongest link' argument. Since these kinds of collective goods can be Pareto optimally produced by a sufficient number of actors, the best prospect for producing the good is to make sure that only a sufficient number of actors is maximally interested in producing the good.

Roughly, we conclude that negative effects of heterogeneity are to be expected in social dilemmas of the Prisoner's Dilemma type and positive effects of heterogeneity in social dilemmas of the Volunteer's Dilemma type. While these predictions are already theoretically well elaborated for the second class of dilemmas, this is not the case for the first class. Thus, the theoretical contributions of this paper are concerned primarily with social dilemmas of the Prisoner's Dilemma type, namely Support Games.

2. Dyadic Social Exchange among Heterogeneous Actors

Social support takes place in different situations and among different people. Common examples are persons helping a neighbor in the garden, children playing with the toys of other children, women exchanging cooking books, and colleagues offering advise or helping out to meet deadlines (Blau 1968; Homans 1961; 1958). Why would people be willing to support other actors, i.e., use their resources for the aims of others rather than for their own aims? Following social exchange theorists such as Homans and Blau, we assume that social support, as other human actions, are based on rewards and costs. Supporting a stranger, likely, is not motivated by rewards obtained from the receiving actor in return. To explain why support is given to a stranger we would have to point to factors such as social norms or psychological processes such as upholding self-esteem etc. (see, e.g., Baurmann 2002; Brennan/Pettit 2004; Frank 1989). Typically, however, social support occurs in the context of *durable pairwise* relationships (Emerson 1976). In a durable relationship, it can be individually rational to provide support because the receiving other actors may repay the support in the future. If Alter is not helping Ego today, Ego might not help Alter either next time. In this sense, the long-term benefits of supporting each other can be higher than the short-term benefits from refusing support. Thus, support in durable relationships is based on the possibility that actors can threaten each other with refusing support and actors can promise each other rewards for being supported. This mechanism is known as *reciprocity* (Axelrod 1984). We do not consider 'generalized reciprocity', i.e., exchange of support amongst more than two actors (see, e.g., Gouldner 1960; Yamagishi/Cook 1993).

Various factors influence these long-term benefits: First, there is a 'time-lag' between providing and receiving social support, creating a trust problem (Coleman 1990, chapter 6). Second, people tend to value future rewards less than present rewards (Loewenstein/Elster 1992). Finally, the rewards in social support are to some extent unspecified (Blau 1964). Alter can never be sure what his or her future reward will be. For instance, if Alter takes time to explain Ego how to use a new computer program, Ego might reward Alter later with an invitation for dinner of unspecified quality.

The interaction structure of social support is similar to a (repeated) Prisoner's Dilemma (Weesie 1988). Each actor has an incentive not to provide support, but prefers to receive the benefits from being supported. Mutual support is efficient if the benefits derived from support received exceed the costs for providing support. The payoffs of Prisoner's Dilemma may be asymmetric, but the Prisoner's Dilemma by itself does not provide a simple interpretation how asymmetric payoffs may arise from individual differences in preferences or resources. The social support interpretation makes it possible to link individual characteristics to the payoffs. The relevant characteristics of actors are the costs for providing support (c_i), the benefits from being supported (b_i), and the probabilities of needing support (π_i) (see Hegselmann 1994a; 1994b; Weesie 1988). In the social support variant of social dilemmas, we can conceptualize the heterogeneity of actors as the dissimilarity of the actors with respect to these characteristics. In an asymmetric Prisoner's Dilemma, the payoffs are heterogeneous. We can now address how the individual characteristics affect whether support is being given. We can also analyse how heterogeneity at the individual level, i.e., heterogeneity in individual characteristics, affects the outcome at the dyadic level, i.e. exchange of support between two actors.

The comparison of exchange of support among heterogeneous actors with a situation of exchange of support among homogeneous actors, however, is not straightforward. If we want to determine whether heterogeneity facilitates or hampers cooperation, we need to devise a method to link up different situation. With what homogenous situation do we want to compare a given heterogeneous situation? If in the homogenous situation all actors have lower costs of providing support than in the heterogeneous situation, it will come as no surprise that exchange of support is more likely under homogeneity. But such a comparison is not very meaningful, as it mixes up two aspects: the 'size' of the pie and the 'distribution' of the pie. The size of the pie refers to the sum of the probabilities that actors need support, and to the 'sum' of the incentives. The distribution of the pie refers to the distribution of the total probability of needing support over the actors and the distribution of the incentives over the actors. The analysis of the consequences of heterogeneity is only meaningful if we keep the 'size of the pie' fixed. Thus, we keep the 'total amount' of the costs, benefits, and neediness of support fixed and study the consequences of varying the distribution of the parameters among the actors. This resembles the distribution of a fixed amount of tangible resources among the actors. Thus, we interpret the parameters c, b, and π as a function of resources. Similar approaches to study effects of 'inequality' can be found, for instance, in the literature on income inequality (see, e.g., Atkinson 1983; Sen 1997) and on insurance and uncertainty (see Arrow 1951).

Social support is intensively studied, both theoretically (Kelley/Thibaut 1959; Homans 1958; 1961; Blau 1964) and empirically (Fischbacher et al. 2002; Ben-Porath 1980; Kirmeyer et al. 1987; Robertson et al. 1991). This literature proposes many hypotheses that are related to our research. For instance, mutual support in a durable relationship is more likely, if the benefits for being supported are higher, if the costs are lower, and if the relation is more durable (Brown 1986, 55ff.; Smith/Mackie 1995, chapter 12). Such hypotheses address individual characteristics, however, these are not the focus of our study. The degree of heterogeneity of actors is a dyadic characteristic, and this is intricately related to the (dis)similarity of actors. It is often found in the literature that social support is more frequent found among similar actors. The interpretation of this finding from the perspective of our model and from our results is discussed in section 5.

3. A Model of Support

3.1 Description of the Model

We consider an interaction with two actors (see Figure 1). At any time point, actors may need the support of the other actor. More precisely, at each time $t$one out of four events may occur: (1) actor 1 needs support, but actor 2 does not; (2) actor 2 needs support, but actor 1 does not; (3) neither actor 1 nor actor 2 needs support, and finally (4) both actors need support. If only one actor needs support, the other actor has two behavioral alternatives, namely to provide support or not to provide support. Providing support costs resources, c_i, receiving support is beneficial, b_i. The parameters b_i and c_i are utility *differences*, not utilities. The benefits b_i are the differences among i's utilities, if i needs and receives support $(x_i + b_i)$, and i's utilities if i needs but does not receive support (x_i). The costs c_i are the difference among i's utilities, if i does not need support and does not give support (y_i), and i's utilities if i does not need but gives support $(y_i - c_i)$. We assume that the benefits from receiving support are larger than the costs for providing support, $b_i > c_i$, with $i = 1, 2$. This assumption reflects the situation that social support comprises the use of one's time to assist another actor reaching his or her goals (Coleman 1984). Of the remaining events, one is simple: if neither of the actors needs support, no support has to be given. This event is valued y_i. If both actors need support at the same time, we assume that no support can be given; this event is valued x_i. Our results only depend on b_i, and c_i. The behavioral choices of i are irrespective of x_i and y_i. Without loss of generality, we can simplify the presentation of our model and write $x_i = y_i = 0$. We only assume that $b_i > c_i > 0$, with $i = 1, 2$.

Consider the example of two colleagues being at work on Friday afternoon. Alter needs help, to put 1400 questionnaires in envelopes in the next four hours before the post office will be closed. Alter's colleague, Ego, has no pressing duties. In this situation, exactly one actor needs help and the other can decide whether to help or not. If Alter puts the questionnaires in the envelopes alone, Alter may not finish the work in time. If Ego helps Alter, Alter will finish work in time. The situation is different if both Ego and Alter have to put 1400 questionnaires in envelopes in the next four hours. Now they cannot help each other. Obviously, if neither Alter nor Ego has to put questionnaires in envelopes, none of them needs help.

In our model it is determined at random which of the four events occurs. Denote by π_i the probability that only actor i needs support at t, by π_0 that no actor needs support at t, and by π_{12} that both actors need support at t. Thus,

$$\pi_1 + \pi_2 + \pi_0 + \pi_{12} = 1, \text{with } \pi_1, \pi_2 > 0.$$

Thus, with probability $\pi_1 + \pi_2 = \pi_+ > 0$ exactly one actor needs support, while the other actor has to make a decision whether or not to provide support. How often support is needed and could also be given (π_+) depends on different aspects of the interdependency situation. For instance, if an advertising agency has acquired a major assignment, all employees will be involved and might need some help from colleagues to get their part done in time. In this case, the probability π_{12} that Alter and Ego need help at the same time will be relatively high. However, if Alter is doing the creative part of the work in the early phase of the project, while Ego has to present the results to the client later on, Alter and Ego do not need help at the same time. At the beginning of the project is the probability π_1 that Alter needs support high, and at the end of the project is the probability π_2 that Ego needs help high; the probability that they need support at the same time (π_{12}) is low. During summer, the workload is low and the probability π_0 that no worker needs help is relatively high.

The formulation above with four different events encompasses the models proposed by Hegselmann (1994a; 1994b) and Weesie (1988). In Weesie (1988), always exactly one actor needs support. Hegselmann assumes that the actors need support independently in the sense of probability theory. An example of such independence, consider the need for emotional support among friends. Ego and Alter require emotional support because of events that happen in their respective lives, at work, in relationships with other people, etc. Such events will be only loosely coupled; the occurrence of events for Ego will typically not depend much on what happens in the life of Alter. In our model, as in Weesie (1988), the need of support can be dependent among actors. As an example with dependency in the need of support, consider two neighboring farmers. After a long period of bad weather, a farmer will need the material support of other people to survive. However, the two neighbors tend to need support at the same time, and they will probably not be able to provide support. Informal support is not very efficient for 'positively' correlated risks; formal insurance schemes are able to pool risks that are geographically more dispersed. These risks are less correlated, and so in this case a formal support mechanism have an efficiency edge over informal support.

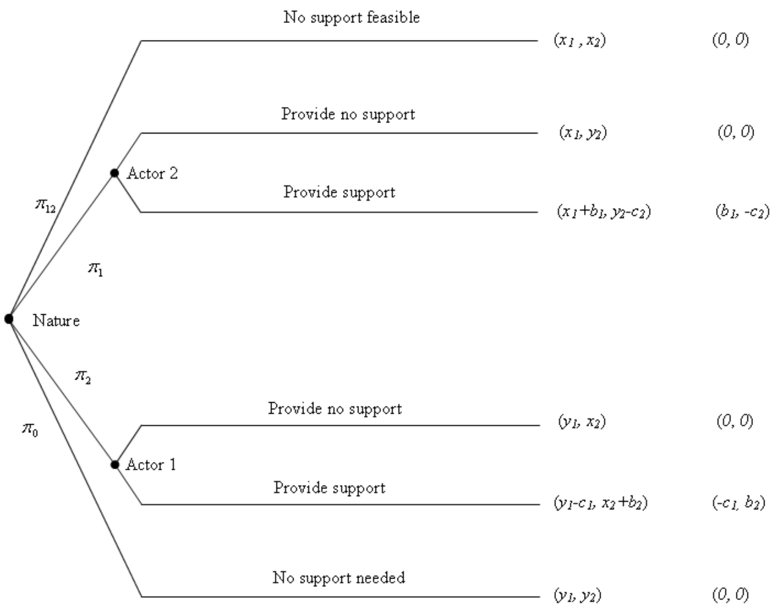


Figure 1: Extensive Form of SG, with $b_i > c_i > 0$. The equilibrium behavior does not depend on (x_i, y_i). Without loss of generality we write $x_i = y_i = 0$.

Figure 1 shows the extensive form of the Support Game (SG). Payoffs represent 'utilities' that correspond to the outcomes of the game. We assume that actors know the game, know that both of them know the structure of the game they play, etc; technically, the structure of the game is common knowledge (Rasmusen 1994). The game is played non-cooperatively in the standard game theoretic sense that actors are unable to make enforceable agreements or commitments. We want to study necessary and sufficient conditions for support in SGs: Under what conditions is it individually rational for the actors to provide support? The results for SG are actually trivial: There is a unique (subgame perfect) Nash equilibrium such that both actors do not provide support. This follows from the assumption that $c_i > 0$. This equilibrium is Pareto inferior compared to the situation in which *both* actors provide support in the events support is needed by exactly one actor, if and only if the expected gains from received support exceed the expected costs from providing support,

$$\pi_i b_i - \pi_j c_i > 0 \text{ for } i = 1, 2,\ j \neq i. \tag{1}$$

Throughout this paper (1) is assumed. If (1) is met, individually rational behavior leads to a collectively irrational outcome (Rapoport 1974).[1] Under condition (1), the SG is a variant of the Prisoner's Dilemma game. It is a Pareto improvement for both actors to provide support instead of not to provide support, but it is individually rational not to provide support.

As discussed in section 2, social support in a durable relationship is based on the principle of reciprocity. Hence, social support has to be studied not in a 'one-shot' game, but in the context of durable relationships: an Iterated Support Game (ISG). The example of the two colleagues provides some intuition why exchange of social support has to be modeled as an ISG. Giving help and advice among colleagues is not controlled or organized by formal contracts among the colleagues. The underlying mechanism of mutual support is reciprocity: A colleague who provides support loses leisure time or might run into a delay with his or her own work. However, the long-term benefits can overcome Alter's short-term incentive not to offer own working time to help Ego. Next time, when Alter needs help, Alter will be better off if Ego rewards Alter for being helped before instead of refusing help. 'Repeated' or 'iterated' games are games in which actors have to make similar choices repeatedly and they can take into account what has happened in previous periods of play (Fudenberg/Tirole 1991). It is well known that if a one-shot game with an inefficient outcome is repeated an indefinite number of times, it can be individually rational to play efficient behavior, provided that the future is 'important enough' and deviations from Pareto optimal behavior are appropriately sanctioned (Friedman 1986). In our application, an efficient outcome means that support is provided whenever support is needed and giving support is possible, i.e., in events 1 and 2.

Let us now address the issue that the future should be important enough. In the theory of repeated games it is usually assumed that actors value the payoffs from the present game higher than the payoffs from future games. There are two reasons for the preferential treatment of the present. The first reason is negative time preferences: People prefer to have a benefit immediately, rather than with a delay (Loewenstein/Elster 1992). A natural way to take this into account is to discount payoffs over time such that the payoffs from the next round are less worth than the one from the current round. The 'weight' actor i assigns to the next round relative to the present one is called i's *discount parameter*

$$\theta_i, \text{ with } 0 < \theta_i < 1, i = 1, 2.$$

Empirical evidence suggests sizable interpersonal differences in discounting that are, at least partly, socially produced (Gattig 2002).

[1] It can be shown that, even if (1) does not hold, there is always a Pareto improvement in which at least one actor j always provides support $\alpha = 1$, while the other actor i provides support with some probability $\alpha_i, 0 < \alpha_i \leq 1$.

The second reason to down weight the future is that future games may not be played after all, because the relation has terminated. We thus distinguish the time discount parameter θ_i from the probability that after each round another round will be played, the continuation probability

$$w, \text{ with } 0 < w < 1.$$

We stress that the 'objective' continuation probability w is the same for both actors, while the 'subjective' individual discount parameter θ_i may well differ between the actors.

The payoffs of the repeated game can now be defined as follows: In every time period t, actor i obtains a payoff u_{it}. Here, $t = 0, 1, 2, ...$ is a discrete time parameter, and $i = 1, 2$ indexes the actors. At each period t in which actor i does not play, he or she obtains a zero payoff, $u_{it} = 0$. The expected discounted payoff is the total payoff of an actor i associated with the infinite stream of payoffs $(u_{i0}, u_{i1}, u_{i2}, ...)$

$$\sum_{t=0}^{\infty} \theta_i^t E u_{it},$$

where Eu_{it} is the expected utility for actor i at time t. Eu_{it} depends on the continuation parameter w, the strategies used by the players, and the benefits, costs, and probabilities of needing support. For instance, if both players always provide support, the expected payoff attained at time t equals

$$Eu_{it} = w^t (\pi_i b_i - \pi_j c_i) + (1 - w^t)0 = w^t (\pi_i b_i - \pi_j c_i).$$

Here w^t is the probability that the relationship has continued up to time t. Thus, the expected discounted payoff obtained over the full interaction if both actors "always provide support" is

$$\sum_{t=0}^{\infty} \theta_i^t E u_{it} = \sum_{t=1}^{\infty} \theta_i^t w^t (\pi_i b_i - \pi_j c_i) = \frac{\pi_i b_i - \pi_j c_i}{1 - w\theta_i}.$$

The expected discounted payoff if both actors never provide support is 0. Expected discounted payoffs for other strategies can be derived in similar ways. Observe that the discount rate commonly included in repeated games models is now the product $w\theta_i$. The distinction between the discount parameter θ and the continuation probability w allows us to address how support in the ISG depends on the 'stability of the relation' indicated by the continuation probability w as well as the individual discount parameters θ_i.

The ISG is similar to games studied in the literature (Weesie 1988; Hegselmann 1994a; 1994b). In Weesie (1988), the focus is, as in this paper, on the dyad level and so on support generated by reciprocity, but differences in the time-preferences are not taken into account. Hegselmann (1994a; 1994b) differentiates actors only by the probability of needing support, but they are able to search for a reasonable partner to exchange support with. Thus, in Hegselmann (1994a; 1994b) support is the consequence of market like social processes.

3.2 Analysis of the Model

Now we can derive the condition for mutual support in ISG. We focus on conditions under which actors are expected to support each other fully and at all time points. Thus, as the term is used here, 'exchange of social support' means that both actors always provide support to each other. However, support will be given only if the long-term benefits of mutually providing support are higher than the short-term benefits of refusing support. The costs of providing support in the long run depend on the strategies used by the actors. We restrict our attention to trigger strategies. Trigger strategies are a particular implementation of reciprocity. A trigger strategy is a *totally unforgiving* strategy that employs *permanent retaliation.* It is never the first to refuse support, but if the other refuses support even once, a trigger strategy refuses support from that time onwards. Even after an own 'unintended' refusal of support, trigger strategies refuse support from then on. This is needed to ensure subgame perfection (Binmore 1998; Kreps 1990).
We do not claim that people in fact use trigger strategies. Furthermore, trigger strategies are not very suitable templates for the study of the dynamics of social strategies. Even though, the analytical treatment of trigger strategies is relatively easy, they do serve an important theoretical purpose. Trigger strategies are theoretically interesting for the study of the preconditions of cooperation. If exchange of social support is not individually rational among trigger strategies, then exchange of social support is not individually rational among other strategies as well. This is the case because in this game permanent retaliation is the most severe punishment (Abreu 1988). The following lemma states the necessary and sufficient condition for an equilibrium in trigger strategies.

Lemma 1. A pair or trigger strategies (τ_1, τ_2) is a subgame-perfect equilibrium if and only if

$$\zeta^* = \max(\zeta_1, \zeta_2) \leq w, \tag{2}$$

where

$$\zeta_i = \frac{1}{\theta_i} \frac{c_i}{\pi_i b_i + (1 - \pi_j) c_i} = \frac{1}{\theta_i} \frac{1}{\pi_i \eta_i + (1 - \pi_j)}, \text{ with } \eta_i = \tfrac{b_i}{c_i} > 1.$$

Proof Lemma 1. For a proof of this lemma and the subsequent theorems we refer to the appendix.

We observe that the equilibrium depends on the costs and benefits via the benefit-cost ratio's denoted by $\eta_i = \frac{b_i}{c_i} > 1$. We take the ratio, since the benefit-cost ratio is a ratio of two utility differences, and so it is a scalar scale free quantity. Thus, interpersonal comparison of the preference parameters η_1 and η_2 is possible without indulging in the intricate problems of the interpersonal comparison of utility (Harsanyi 1977, chapter 4; Sen 1997; Coleman 1990, chapter 29). The equilibrium condition shows that exchange of support depends on the dyadic continuation probability (w) and on all individual parameters: The benefit-cost ratio's (η_i), the time-preferences (θ_i), and the probabilities of needing support (π_i). We will use this condition to predict the frequency of support under homogeneous and heterogeneous distributions of the parameters.

It is an important analytical result of the theory of repeated games, that cooperation is consistent with individually rational behavior in a repeated game if the continuation probability w is sufficiently large (Kreps 1990). This result is replicated here. According to (1), it is individually rational to provide support if the continuation probability w exceeds the threshold ζ^*. If ζ^* decreases or w increases, the equilibrium condition 'is more easily met'. We will interpret this to mean that 'support is more likely' if ζ^* decreases. Keeping w fixed, ζ^* will then be treated as an indicator for 'how likely the exchange of support' is among the actors. The threshold ζ^* depends on the individual thresholds ζ_i. The individual threshold ζ_i increases in the benefit-cost ratio's η_i (i.e., increases in the costs and decreases in the benefits), and in the probabilities of needing support π_i, but decreases in the time preference θ_i. If ζ_1 or ζ_2 increases, ζ^* increases as well. Analogously, ζ^* decreases if ζ_1 and ζ_2 decrease. Therefore, $\zeta^* = \max(\zeta_1, \zeta_2)$ increases in η_i, and decreases in π_i and θ_i for both actors. This result is in accordance with the empirical findings on social support mentioned in section 2.

3.3 Heterogeneity among the Actors

We model heterogeneity of the actors as interpersonal differences in the benefit-cost ratio's (η_i), in the probabilities of needing support (π_i), or in the time preferences (θ_i). Actors are called homogeneous if they do not differ with respect to the three individual parameters,

$$\eta_1 = \eta_2, \quad \pi_1 = \pi_2, \text{ and } \theta_1 = \theta_2,$$

otherwise the actors are said to be heterogeneous. The continuation probability w is always the same for both actors. For instance, if Alter stops working at a company, the work-relation between the colleagues Ego and Alter stops for both of them.

We argue, that a comparison of a homogeneous with a heterogeneous situation is meaningful if the situations are related to each other so that the 'total amounts' of the parameters are fixed

$$\eta_1 + \eta_2 = \eta_+, \;\; \pi_1 + \pi_2 = \pi_+, \;\; \text{and } \theta_1 + \theta_2 = \theta_+. \tag{3}$$

This is our *constant-sum condition* or *budget constraint*. Under this condition, we cannot reduce the individual parameters of both actors at the same time. If we decrease one individual parameter, for instance, the benefits of actor 1, we have to increase the benefits of actor 2 so that the sum of the benefit-cost ratio is not affected ($\eta_1 + \eta_2 = \eta_+$). To keep the sums η_+, π_+ and θ_+ fixed resembles the distribution of a fixed amount of a tangible resource among the actors. The interpretation of a distribution of a tangible good seems appropriate for the benefit-cost ratios and for the probabilities that actors need support. Differences in costs and benefits and in the probability to need support reflect differences in resources. The interpretation is admittedly less compelling for the psychological time preferences. To give a numerical example, we assume a homogeneous distribution of all parameters, except of the distribution of the costs, which is heterogeneous. If we assume that $\eta_1 + \eta_2 = 6$ we can compare the homogeneous situation $\eta_1 = \eta_2 = 3$, with the heterogeneous situations $\eta_1 = 4, \eta_2 = 2$ or $\eta_1 = 5, \eta_2 = 1$. Thus, we can study effects of an increase of heterogeneity in the benefits-costs ratio. Heterogeneity is conceived as an unequal distribution of at least one parameter $(\eta_+, \pi_+, \theta_+)$ among the actors. The equilibrium condition is least restrictive if ζ^* is minimal under every possible distribution of the individual level parameters, given the budget constraint (3). We use the phrase 'distribution' of a parameter to stress that changes at the individual level should not violate the constant-sum condition (3) that ensures that a comparison of situations is indeed meaningful. Now, the minimization of ζ^* is not equivalent to minimizing ζ_1 and ζ_2 at the same time. Given the budget constraint, if ζ_1 decreases, ζ_2 necessarily increases.

3.4 Theorems

To illustrate how the exchange of social support is affected by different distributions of the parameters among the actors, we consider a numerical example with homogeneity with respect to the benefit-cost ratio η and the time discounting parameter θ,

$$\eta_1 = \eta_2 = 3, \text{ and } \theta_1 = \theta_2 = 1.$$

First, we consider homogeneity with respect to the need of support, $\pi_1 = \pi_2 = \frac{1}{2}$. According to Lemma 1 support can be expected if the continuation probability w is larger or equal than the threshold

$$\zeta^* \quad = \quad \max\left(\frac{1}{1}\frac{1}{\frac{1}{2}\frac{9}{3} + (1 - \frac{1}{2})}, \frac{1}{1}\frac{1}{\frac{1}{2}\frac{9}{3} + (1 - \frac{1}{2})}\right) = \max(0.5, 0.5) = 0.5.$$

Next, we consider heterogeneity with respect to need, $\pi_1 = \frac{1}{3}$ and $\pi_2 = \frac{2}{3}$. In this case the threshold ζ^* for individual rational exchange of support is

$$\zeta^* \quad = \quad \max\left(\frac{1}{\frac{1}{3}\frac{9}{3} + (1 - \frac{2}{3})}, \frac{1}{\frac{2}{3}\frac{9}{3} + (1 - \frac{1}{3})}\right) = \max(0.75, 0.37) = 0.75.$$

Hence, the equilibrium condition is more restrictive in the heterogeneous case than in the comparable homogenous case. Figure 2 shows how the individual thresholds ζ_1 and ζ_2, and the 'collective' threshold ζ^* vary with π_1. The figure shows that ζ^* is minimal if the individually thresholds are equal, $\pi_1 = \pi_2 = \frac{1}{2}$, i.e. exchange of support is 'most likely' under homogeneity.

To provide some intuition why homogeneity helps, consider the analogy we discussed in the introduction. We want to make a chain as strong as possible, using the available iron and carbon. With two links and equal amounts of iron already dedicated to produce each of the links, how should the carbon be allocated? The strength of the chain is that of the weakest link, as the weakest link is the first to break under pressure. Clearly, we should make both links equally strong. An unequal distribution of carbon would make one link stronger than the other. Since the chain is as strong as its weakest link, the extra carbon used in the stronger link is wasted; the chain is not optimal. The chain would

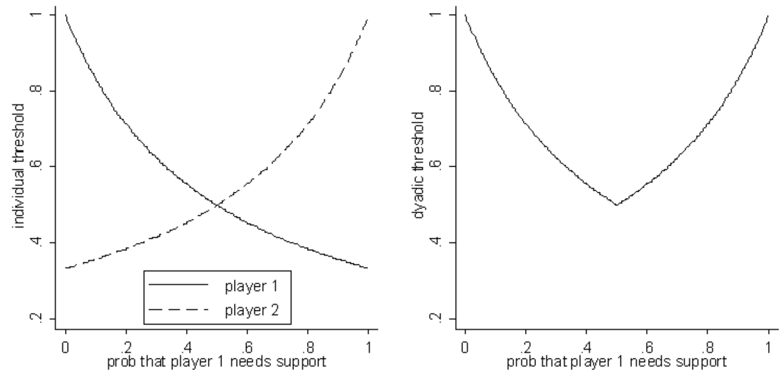


Figure 2: Numerical example of theorem 1 'Heterogeneity in one dimension'

become stronger if some carbon is shifted from the more affluent link to the link poorer in carbon. In an optimal chain, assuming the iron was distributed equally, the carbon has to be distributed equally as well.

The following theorem states that the example above generalizes beyond the arbitrary numbers of the example, and also applies to heterogeneity with respect to the other individual level parameters.

Theorem 1 'Heterogeneity in one dimension': *Consider an ISG with heterogeneity in one parameter $\mu \in \{\eta, \pi, \theta\}$ and homogeneity in the remaining two parameters. Then ζ^* is minimal if μ is distributed equally among the actors, and so $\zeta_1 = \zeta_2 = \zeta^*$.*

Homogeneity is best if we allow heterogeneity only in a single dimension. Assume that Alter and Ego are colleagues. From time to time Ego and Alter have to run experiments. Then they need the help from each other—if they would run the experiments at the same time, they could not assist each other. If Alter assists Ego, Alter loses time, if Ego assists Alter the situation is the same just the other way around. If both actors require help from each other equally often the prospects for exchange of support are optimal.

In our first example, we illustrated the optimal adjustment of one parameter if the other two parameters are homogenous. Now we consider an example with heterogeneity with respect to the costs and benefits and homogeneity with respect to time discounting.

$$\eta_1 = \tfrac{12}{3}, \eta_2 = \tfrac{6}{3}, \quad \text{and } \theta_1 = \theta_2 = 1.$$

We want to study whether or not homogeneity, with respect to the probability to be in need, is still optimal. First, we consider again the homogeneous case, $\pi_1 = \pi_2 = \frac{1}{2}$. Now the equilibrium threshold becomes

$$\zeta^* \quad = \max\left(\frac{1}{1}\frac{1}{\frac{1}{2}\frac{12}{3} + (1-\frac{1}{2})}, \frac{1}{1}\frac{1}{\frac{1}{2}\frac{6}{3} + (1-\frac{1}{2})}\right) = \max(0.4, 0.7) = 0.7.$$

Next, we consider a particular heterogeneous distribution of need, $\pi_1 = \frac{1}{3}$ and $\pi_2 = \frac{2}{3}$. In this case we have:

$$\zeta^* \quad = \max\left(\frac{1}{1}\frac{1}{\frac{1}{3}\frac{12}{3} + (1-\frac{2}{3})}, \frac{1}{1}\frac{1}{\frac{2}{3}\frac{6}{3} + (1-\frac{1}{3})}\right) = \max(0.6, 0.5) = 0.6.$$

Thus, we see that in this example homogeneity in need does *not* lead to the minimum of ζ^*, i.e., equal need of support is not the most favorable condition for exchange of support. The particular heterogeneous distribution of the π's that we looked at leads to a smaller threshold than a homogeneous distribution of the π's. It is, however, not the heterogeneity of the π's per se that facilitates exchange of support. If the probability distribution is oppositely skewed, namely $\pi_1 = \frac{2}{3}$ and $\pi_2 = \frac{1}{3}$, ζ^* would be even larger than under homogeneity:

$$\zeta^* \quad = \max\left(\frac{1}{1}\frac{1}{\frac{2}{3}\frac{12}{3} + (1-\frac{1}{3})}, \frac{1}{1}\frac{1}{\frac{1}{3}\frac{6}{3} + (1-\frac{2}{3})}\right) = \max(0.3, 1) = 1.$$

Since $0 < w < 1$, exchange of social support is not individually rational under this condition, no matter how durable the relationship is. If $\pi_1 = \frac{1}{3}$ and $\pi_2 = \frac{2}{3}$, there is a compensation effect between the parameters: Heterogeneity in one parameter (η) is *compensated* by heterogeneity in another parameter (π). Figure 3 displays the individual thresholds (ζ_i) and the dyadic threshold (ζ^*) for varying π_1. We see that exchange of support is 'most likely' under a heterogeneous distribution of the probabilities of needing support. If $\pi_1 = \frac{1}{3}$ and $\pi_2 = \frac{2}{3}$, the individual thresholds are equal and the dyadic threshold has the smallest value. If$\pi_1 = \pi_2 = \frac{1}{2}$, ζ^* is larger, and furthermore we see that it is not the heterogeneity of the π's per se that facilitates exchange of support, because ζ^* is even larger if $\pi_1 = \frac{2}{3}$ and $\pi_2 = \frac{1}{3}$. In this latter case, the heterogeneity in the two parameters re-enforces each other.

We conclude that homogeneity in a given parameter is not necessarily 'optimal', irrespective of the distributions of the other parameters. If there is heterogeneity in one parameter, it is usually the case that better prospects for the exchange of support are possible if there is heterogeneity in the other dimension as well, provided that these heterogeneities are well aligned.

Theorem 2 'Compensation': *Consider an ISG with heterogeneity in η, π, θ. ζ^* is minimal in one of the parameters $\mu \in \{\eta, \pi, \theta\}$ if μ_1 and μ_2 are adjusted so that $\zeta_1 = \zeta_2 = \zeta^*$.*

While generally it is not true that homogeneity with respect to π, or any other parameter for that matter, favors exchange of support, some form of homogeneity is beneficial, namely the *theoretical homogeneity* in terms of the individual thresholds ζ_i. These are the theoretical analogues to the 'strength of the link'. Considering the numerical example, but now with general probabilities π_1 and $\pi_2 = 1 - \pi_1$, we have

$$\zeta_1 = \frac{1}{1(\frac{12}{3}\pi_1 + (1-\pi_2))} = \frac{1}{5\pi_1} \qquad \zeta_2 = \frac{1}{1(\frac{6}{3}\pi_2 + (1-\pi_1))} = \frac{1}{3(1-\pi_1)}$$

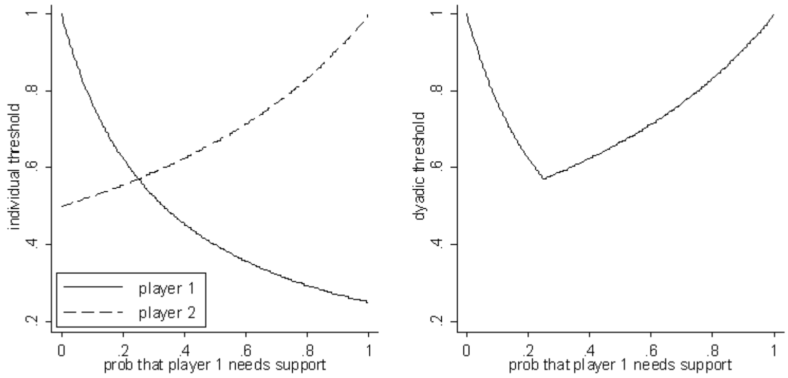


Figure 3: Numerical example of theorem 2 'Compensation'

According to theorem 2, the optimal ζ^* is characterized by the condition $\zeta_1 = \zeta_2$. Solving for π_1 gives $\pi_1 = \frac{3}{8}$, and so the optimal threshold $\zeta^* = \frac{8}{15} = 0.533$.

We consider again the example of the two colleagues who help each other assisting experiments. We assume that Alter and Ego have equal benefits from receiving help. However, Ego runs experiments more often than Alter does. The optimal prospect for mutual help would then be that Ego's experiments are shorter than Alter's experiments, thus, Alter has lower costs for providing help than Ego. The low costs for providing support compensate the high frequencies of running the experiments (compensation effect).

So far, we analyzed how *one* parameter (μ) is optimally distributed, given how the other two parameters are distributed. We now analyze variations in all three parameters at the same time, subject to the constant-sum conditions (3). Each parameter can be distributed homogeneously or heterogeneously. How should the parameters be distributed *simultaneously* in order to make the condition for exchange of support least restrictive? The question is answered in the next theorem.

Theorem 3 'Homogeneity is globally best': Let $\pi_+ = 1$.[2] *The minimum of ζ^* subject to the constant-sum condition is attained by homogeneity with respect to each of the parameters, i.e.,* $\eta_1 = \eta_2 = \frac{1}{2}\eta_+$, $\pi_1 = \pi_2 = \frac{1}{2}$, and $\theta_1 = \theta_2 = \frac{1}{2}\theta_+$

In the chain analogy, all material should be distributed equally among all links to make the chain as strong as possible. This is what happens if a chain out of steel will be produced: Given some amounts of iron, zinc and copper, and no constraints how the elements should be distributed to produce the chain, every link of the chain will have the same amount of iron, zinc, and copper. With respect to the example of the two colleagues the optimal prospect for mutual support would be that both have same costs and benefits for assistance

[2] For a discussion of the technical assumption that $\pi_+ = 1$, see the proof of theorem 3 in the appendix.

in conducting experiments and for getting help with the own experiment, and that both need for support equally often, i.e., both run experiments equally often.

4. Discussion and Limitations

In this paper we have argued that the effects of asymmetry in social dilemmas can be conveniently studied in terms of exchange of social support with asymmetry conceptualized as actor heterogeneity with respect to the costs for providing support, the benefits from receiving support, the probability that support is needed by an actor (while the other actor does not need support), and how much actors value future rewards. Our analysis yielded a series of theorems. First, if there is homogeneity in all but one parameter, homogeneity in the remaining parameter leads to the optimal condition for exchange of social support. Second, if there is heterogeneity in at least one parameter, exchange of support is most likely if there is a specific heterogeneous distribution of the other parameters that 'compensates' for the original heterogeneity. Thus, homogeneity in a single dimension does *not* necessarily lead to the optimal condition of exchange of support. Third, if all parameters are varied 'at the same time', exchange of support is most likely if all parameters are distributed equally among the actors. Thus, homogeneity in all dimensions is the globally best outcome.

A good intuition for our results can be obtained from the analogy of how to make a multiple-link chain as strong as possible. This is achieved by making the weakest link as strong as possible, and so we should make all links equally strong. There are however many ways to do this. Intuitively it is not implausible that the chain is indeed strongest if all links are the same. A sufficient reason for the optimality of homogeneity is *convexity* of the strength of links in terms of the amounts of different materials used, and so the different materials need to be *complementary*.

In the social support case, *all* actors need to be willing to stick to the implicit cooperative agreement enforced by trigger strategies. Thus, to facilitate cooperation the *thresholds* of both actors should be as small as possible. We conclude that the individual thresholds have a similar role as the inverse of the strength of the links. Both individual thresholds are related due to the 'fixed mean' of the parameters: a decrease of the individual threshold of Alter can be accomplished only by an increase of the individual threshold of Ego and vice versa. Thus, making both individual thresholds as small as possible implies that the individual thresholds should be equal. To obtain the result that optimal individual thresholds involve homogeneity with respect to the parameters, however, requires additional arguments, just as we needed for the chain analogy. In the proof of theorem 3, we showed that the individual thresholds are indeed convex in the parameters. One can think of this, as a positive interaction effect between any two of the parameters.

We want to emphasize that 'homogeneity is globally best', does not mean that 'less heterogeneity' among actors necessarily yields more cooperation than 'more heterogeneity'. Heterogeneity decreases if the distribution of any of the parameters is made more equal. However, if such a reduction in heterogeneity is performed in a parameter that was skewed to compensate for inequality in another parameter, the compensation effect will become smaller, and so cooperation will be hampered, not facilitated. Numerical examples of this phenomenon were presented in section 3. We turn again to our analogy for illustration. Making the distribution of the construction materials over links more similar does not necessarily make the chain stronger. Consider the example of a chain made out of copper and iron. The iron is distributed equally among all the links of the chain, except for one link that has only half the amount of iron. If we employ the copper symmetrically, the link with less iron will still be the weakest link. If we put somewhat more copper in the link low in iron than in the other links, i.e., we use the materials less homogenously and we do improve the strength of the chain.

This discussion may shed new light on the research of similarity in social support theory. Theories of social support often argue that 'similarity' between actors influences positively supportive behavior. The general idea is that in deciding to help or support, people take into account how similar the person in need is (Smith/Mackie 1995). The theories claim that similarity generally enhances cooperative tendencies such as conflict resolutions, supportive behavior etc.: "Similarities in beliefs, attitudes, and values ... are usually conducive to ... cooperative resolutions of conflicts." (Deutsch 1973, 374) "Similar others usually end up receiving the most help." (Dovidio et al. 1991) Those theories usually study effects of heterogeneity and homogeneity in variables such as age, gender, religion, social status, or education on social support. It is very seldom that the effects of *different degrees* of homogeneity and heterogeneity on support *between* several independent variables such as age, gender, etc. are compared with each other.

As an example of an article that uses homogeneity cumulatively we discuss Louch's article on network integration (Louch 2000). The article combines studies of transitivity and homophily (homogeneity in our terms) in an empirical analysis of personal network integration. For the purpose of our demonstration we focus only on the homophily hypothesis and neglect the other hypotheses. This states that homophily improves the likelihood of integration in personal networks (triads). Louch uses race, gender, education, age, age^2, and religion to test this hypothesis. Using a logistic regression model Louch analyses the simultaneous effects of all variables on the likelihood of a connection existing between triads. Louch is using the different homophily variables in this model additively. The effect of each homophily variable is namely studied by keeping the other homophily variables constant. The effect of age is, for instance, studied by keeping education, religion, gender, and age constant. Given the third theorem on compensation, we would expect an interaction effect between homophily variables. Since the difference in homogeneity and heterogeneity *between* the variables *per se* has an effect on supportive behavior a cumulative analysis of homophily variables is misleading. The compensation theorem states a complex interaction effect between heterogeneous variables.

Take as an example two colleagues assisting each other by running experiments. Now we can have the situation that Alter runs experiments more often than Ego. That means that Ego has to assist Alter more often than Alter assists Ego. If the experiments of Ego and Alter take the same time, then we have a situation where Ego has to help Alter more often and each time when Ego helps it takes quite long. It is easy to imagine that Ego will not be willing to assist Alter all the time. However, if the experiments of Ego take much longer than the experiments of Alter, then the situation is different. Now, Ego still has to help Alter more often, but since Alter always assists Ego longer than Ego assists Alter, the situation might be more equal than in the first situation. Only looking at differences in each of the variables separately, one would expect that heterogeneity in frequencies as well as in the length of the experiments make helping each other more problematic.

However, it is clear for this example that the main predictor for helping behavior should be the total time each person needs to be helped, which is given by the interaction between frequency and length of the experiments. In other words, if one needs more often help it is less costly for the others to help, if the heterogeneity in one dimension compensated the other. However, this interaction between the variables is often neglected in the literature on homophily. The implicit assumption of most of the homophily analyses it is generally better to have *more* dimensions that are homogeneous is not necessarily true. Given heterogeneity in one dimension, it can be even better for exchange of support to have even *more* heterogenous dimensions. Heterogeneity in one dimension can namely be compensated by heterogeneity in another dimension. However, the positive side of heterogeneity, namely compensation, is mostly ignored in the literature on similarity or homophily.

We like to stress that *compensation* has nothing to do with *complementarity*. Complementarity may also influence the parameters of the game. Kelley and Thibaut (1959) emphasize that similar actors may not be *able* to provide support to each other. Complementarity of the actors is an essential precondition for exchange of support. This issue was also apparent in the example of neighboring farmers; actors facing highly positively correlated risks are not complementary. Complementarity of actors leads to relative high probabilities π_i that actors who can provide support are able to do so, because they are not in trouble themselves.

We briefly want to sketch how the hypotheses can be tested with data from a laboratory experiment. We let subjects play series of ISGs. It is sufficient to vary only two dimensions in the experiment to test the hypotheses: the benefits from receiving support (b_i) in the benefits-costs ratio (η_i), and the probabilities of needing support π_i). The costs (c_i) in the benefits-costs ratio (η_i) can be fixed, and the time preferences (θ_i) can be neglected to test the hypotheses. The budget constraints are selected at $\pi_+ = 1$, and $\eta_+ = 6$. Subjects play Support Games with parameters as in one of the four conditions displayed in the 'prediction' table below.

Condition Description	π_1	π_2	$\boldsymbol{c}_1 = \boldsymbol{c}_2$	$\boldsymbol{b}_1 = \eta_1$	$\boldsymbol{b}_2 = \eta_2$	ζ^*
Symmetry in η, π	0.5	0.5	1	3	3	0.50
Asymmetry in π	0.7	0.3	1	3	3	0.83
Accumulation	0.6	0.4	1	4.5	1.5	1.00
Compensation	0.3	0.7	1	4.5	1.5	0.61

The table can be read as follows. The rows contain the conditions. Columns $1 - 3$ list the probabilities of needing support, the costs of providing support, and the benefits-costs ratio from being supported for actor 1 and actor 2. The last column lists the dyadic threshold as derived from the game theoretic model (ζ^*). Those conditions reflect the hypotheses about symmetry, asymmetry, and compensation. Theorem 1 can be tested, by comparing the support rate of the conditions 'symmetry in c, π, b' and 'asymmetry in π'. Theorem 1 states that, instead of asymmetry in one dimension (e.g., asymmetry in π), it is better to have symmetry in all dimensions (e.g., symmetry in η, and π). Theorem 2 states that asymmetry in one dimension can be compensated with asymmetry in another dimension. We can see that under the condition 'compensation' the asymmetry in η_i compensates the asymmetry in π_i. Under the condition 'accumulation', we can see that it is not necessarily the case that asymmetry in one dimension adjusts asymmetry in another dimension. We can test theorem 2 by comparing the support rate in the two conditions. As stated in theorem 3, the smallest threshold of the table is under 'Symmetry in η, π'. Of all conditions, we would expect the support rate to be the highest under full symmetry.

Finally, we want to discuss some limitations of our analysis and possible remedies to overcome these shortcomings. The first issue concerns our focus on all-or-nothing trigger strategies. Actors either provide full support, backed up by the threat to cancel all future support after any misdeed, or actors do not provide support at all. The restriction to these two equilibria seems to some extent unreasonable. We are interested in heterogeneity among actors, but we study equilibria in which all actors behave the same. It seems quite reasonable, however, that the more powerful actor, e.g., the actor with least costs for providing support, provides support less often or to a lesser degree than the one with higher support costs (Blau 1964). Maybe we should consider asymmetric equilibria that typically exist in iterated games (see 'folk theorem' results in, e.g., Kreps 1990; Friedman 1971). These asymmetric equilibria may be Pareto ordered, but may be also Pareto incomparable, leaving a bargaining problem. Who gains how much? To answer the question 'who gains how much', we suggest to study 'terms of trade' which can conveniently take the form of fractional support. In the model with fractional support, actors make decisions with respect to the degree σ_i to which they give support, $0 \leq \sigma_i \leq 1$, with costs c_i and benefits b_i proportional to σ_i (for an analysis of such a 'continuous game', see Nowak et al. 1989; 1999). For instance, either Alter helps Ego to search for participants for an experiment, or Alter only helps to analyze the results, or Alter does both, or Alter does nothing. In our analysis, we fixed $\sigma = 1$. Do we have any good reasons to fix the σ's in a certain way? If not, we face a selection problem of the σ's. We need to address the bargaining problem and study the situation with a bargaining model (Kalai-Smorodinski 1975; Nash 1950; Rubinstein 1982; 1990). However, in our case bargaining theories have a special disadvantage: The cooperation problem is solved 'by assumption', because bargaining theories assume from the outset Pareto-efficiency. As a solution we may further use an evolutionary approach (Holland 1975; Axelrod 1984; Weibull 1995; Hofbauer/Sigmund 1998). By using an evolutionary approach, we provide an answer (a) how likely social support is among heterogeneous actors and (b) which of the heterogeneous actors provides support to what extent.

A second extension of our analysis would comprise *qualitative* differences between situations with homogenous and heterogeneous actors. Such differences refer to social processes that depend crucially on the homogeneity or heterogeneity of the actors. An example is 'symmetry' considerations in *equilibrium selection.* We will illustrate this point with bargaining situations. Here homogeneity may have advantages as well as disadvantages for the actors compared to heterogeneity. In a bilateral bargaining situation with homogeneous actors, an even split is a likely and obvious 'focal point' solution. Focal points are equilibria that are particularly compelling for psychological reasons (Kreps 1990; Schelling 1960). Thus, homogeneity can reduce negotiation costs. If an 'obvious' solution exists, the partners do not have to bargain for it, and there is little chance that such a negotiation fails, and so exchange of social support is likely in the relation. On the other hand, there are also situations where heterogeneity seems to be helpful. An example is a relationship that turned out sour. To improve the relationship, one of the partners has to make an effort to improve the relationship, for instance by starting to give support that is not immediately reciprocated. Here, heterogeneity may be an advantage. The actor who has more to gain from restoring the exchange of support likely takes the initiative without much delay (see Weesie 1993 for a game theoretic analysis). In a homogeneous relation, both actors may well wait and see whether the other actor takes the initiative. A similar study, also related to conflict-management, can be found in the article about theoretical and experimental evidence in the Volunteer's dilemma of Diekmann (1993). The experimental data showed that the more interested actors are more willing to produce the 'collective good' than the less interested actors. These advantages and disadvantages of homogeneity may lead to interesting *non-continuous* effects of heterogeneity in the neighborhood of homogeneity that deserve further scrutiny.

Appendix: Mathematical Details

Proof of Lemma 1. The game analyzed is a repeated game with infinite horizon and exponential discounting. A useful tool for this analysis is Bellmann's optimality principle of dynamic programming (Kreps 1990). Due to this principle, playing a trigger strategy is individually rational if and only if all *one-step deviations* are not profitable, i.e., do not increase payoffs. Without loss of generality, consider a deviation at time $t = 0$, assuming that i has to decide whether to support or not. The expected and discounted payoff if actor i chooses to provide support at $t = 0$ is

$$EU(\text{provide support}) = -c_i + \theta_i((1-w)0 + w\tfrac{\pi_i b_i - \pi_j c_i}{1-w\theta_i}).$$

Here $-c_i$ is the payoff of the first time providing support at time $t = 0$, followed by the expected and discounted payoff for always providing support from $t = 1$ onward. The expected and discounted payoff if i does not provide support at $t = 0$, and hence support is never given among the actors, is:

$$EU(\text{not provide support}) = 0$$

The expected and discounted payoff is 0, because trigger defects forever if the other strategy defects even once or if trigger defects itself. Thus, according to Bellman's optimality principle, a necessary and sufficient condition for an (subgame perfect) equilibrium in trigger strategies is

$$-c_i + \tfrac{w\theta_i}{1-w\theta_i}(\pi_i b_i - \pi_j c_i) \geq 0.$$

Using some straightforward computation this is equivalent to (1).

Proof of Theorem 1 ('Heterogeneity in one dimension'). Without loss of generality, let $\zeta_1 > \zeta_2$. Since ζ_i increases in π_i, it follows that $\pi_1 > \pi_2$. If we 'take away' some amount of π_1 from actor 1, and 'give it' to actor 2, π_2 increases in the same amount we took away from π_1. Hence, ζ_1 decreases and ζ_2 increases, while $\zeta_1 > \zeta_2$ still holds provided the amount transferred from 1 to 2 is small enough. Thus, ζ^* decreases as well. We can make such transfers, and keep decreasing ζ^*, until $\pi_1 = \pi_2$, and hence $\zeta_1 = \zeta_2 = \zeta^*$. Analogous arguments can be given for η and θ. □

Proof of Theorem 2 ('Compensation'). We consider the optimal adjustment of π. Assume $\zeta_1 > \zeta_2$. We observe that ζ_i increases in π_i. If we transfer some amount of π_1 from actor 1, we have to 'give it' to actor 2 due to the budget constraint. Hence, ζ_1 decreases and ζ_2 increases. So ζ^* decreases, provided that the transfer was sufficiently small. We can make such transfers until we made $\zeta_1 = \zeta_2$. Note that, in contrast to the proof of theorem 1, equalization of the ζ's does *not* imply equalization of the π's. Analogous arguments can be given for optimal adjustments of η and θ. □

Proof of Theorem 3 ('Symmetry is globally best'). Optimality of symmetry can be demonstrated by a convexity argument. The individual threshold

ζ of Lemma 1 can be written in terms of the individual parameters $\alpha = (\eta, \pi, \theta)$ and the total probability π_+ that either actor 1 or actor 2, but not both, need support at any time point,

$$\zeta = \frac{1}{\theta}\frac{1}{\pi\eta + 1 - (\pi_+ - \pi)} = \frac{1}{\theta}\frac{1}{\pi(1+\eta) + 1 - \pi_+}.$$

We claim that ζ is convex in α on$A = [0, \eta_+] \times [0, \pi_+] \times [0, \theta_+]$ for any η_+ and θ_+ under the assumption that $\pi_+ = 1$. Since ζ is smooth, it suffices to show that the matrix H of the second order derivative of ζ with respect to α is positive definite on A. By straightforward computation, we have

$$H = \frac{\partial^2\zeta}{\partial\alpha\partial\alpha'} = \begin{pmatrix} \frac{2\pi^2}{\theta\psi} & \frac{\pi}{\theta^2\psi^2} & \frac{-1+\pi+\eta\pi+\pi_+}{\theta\psi^3} \\ \frac{\pi}{\theta^2\psi^2} & \frac{2}{\theta^3\psi} & \frac{1+\eta}{\theta^2\psi^2} \\ \frac{-1+\pi+\eta\pi+\pi_+}{\theta\psi^3} & \frac{1+\eta}{\theta^2\psi^2} & \frac{2(1+\eta)^2}{\theta\psi^3} \end{pmatrix}$$

with

$$\psi = 1 + (1+\eta)\pi - \pi_+ > 0$$

A necessary and sufficient condition for positive definiteness is that all principal minors of H have a positive determinant (Magnus/Neudecker 1988, 24). We have

$$|H_{11}| = \tfrac{2\pi^2}{\theta\psi^3} > 0$$

$$\left|\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\right| = \tfrac{3\pi^2}{\theta^4\psi^4} > 0$$

$$|H| = 2\tfrac{2(1+\eta)\pi+\pi_+-1}{\theta^5\psi^6} > 0 \text{ since } \pi_+ = 1.$$

and so H is positive definite for all $\alpha \in A$. We conclude that ζ is convex on A. Convexity of ζimplies that $\frac{1}{2}(\zeta(\alpha_1)+\zeta(\alpha_2)) \geq \zeta(\frac{1}{2}(\alpha_1+\alpha_2))$for any α_i. It follows that

$$\zeta^* = \max(\zeta(\alpha_1), \zeta(\alpha_2)) \geq \tfrac{1}{2}(\zeta(\alpha_1) + \zeta(\alpha_2)) \geq \zeta(\tfrac{1}{2}(\alpha_1 + \alpha_2))$$

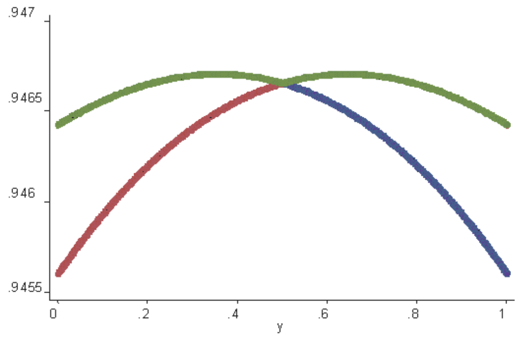
and so ζ^* is minimal under symmetry: 'symmetry is globally best'. □

Remark. Under the assumption that actors discount future rewards at the same rate, $\theta_1 = \theta_2$, it can be shown that symmetry is globally best also if $\pi_+ < 1$, though ζ is not globally convex. The assumption that $\pi_+ = 1$ made in theorem 3 cannot be dropped in general. The assumption is made to ensure that the determinant of the Hessian of the threshold function ζ is positive definite, and the threshold ζ is convex. If $\pi_+ < 1$, the threshold ζ is no longer globally convex. Homogeneity need not be optimal in the part of the parameter space where convexity does not hold. For instance, let

$$\alpha_1 = (\eta_1, \pi_1, \theta_1) = (4.5174, 0.344, 0.9348), \zeta_1 = \zeta(\alpha_1) = 0.9464$$

$$\alpha_2 = (\eta_2, \pi_2, \theta_2) = (3.6751, 0.251, 0.9997), \zeta_2 = \zeta(\alpha_2) = 0.9456$$

$$\overline{\alpha} = (\overline{\eta}, \overline{\pi}, \overline{\eta}) = (4.09625, 0.2975, 0.96725), \overline{\zeta}^* = \zeta(\overline{\alpha}) = 0.9466$$



In this example, homogeneity has a higher threshold than heterogeneity, and so cooperation is harder under homogeneity, but the difference between homogeneity and heterogeneity is very small indeed. This numerical example is illustrative of the counter examples against the claim 'homogeneity is best'. If symmetry is not optimal, it will be negligibly worse than some specific asymmetric distribution. The differences are so small that we don't anticipate that these perverse situations can be identified empirically.

ζ is convex in (η, θ, π), not in $(\frac{1}{\eta}, \theta, \pi)$.

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