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Aspiration Balancing Agreements: A New Axiomatic Approach to Bounded Rationality in Negotiations*

Abstract: A wealth of experimental findings on how real actors do in fact bargain exists. However, as long as there is no systematic general account of the several experiments bargaining theory remains dominated by axiomatic approaches based on normative requirements or on assumptions of full rather than bounded rationality. Contrary to that, the new axiomatic account of aspiration level balancing in negotiations of boundedly rational actors presented in this paper incorporates experimental findings systematically into economic bargaining theory. It thereby forms a descriptive theory of bargaining that has normative power as well.

0. Introduction

Recent bargaining experiments as conducted by experimental economists generally adopt a very austere design. Due to thorough efforts of controlling the situation more often than not there is not much of the flavor of real world negotiations retained. Yet there is an older literature in which the experimental interaction situation is set up in ways more akin to real life. Clearly such a richer design has its disadvantages, too. It reduces the experimental control of variables and makes generalizations often harder to achieve. However, since richer experiments are often much closer to the real world they have very distinct advantages in characterizing real behavior whenever one succeeds in distilling general structures from them. This suggests to go back to the older richer experiments and to re-analyze them for general structures.

There is a nowadays largely neglected German tradition of conducting "realistic" bargaining experiments. In particular people working with the founders of German experimental economics Heinz Sauermann and Reinhard Selten[1] have experimented on bilateral negotiations in a bounded rationality framework. They have described and to some extent analyzed their experiments and they indeed came up with some fairly general insights. For instance the analysis of

* I am indebted to Wulf Gaertner who some twenty years ago offered me the opportunity to start research that brings together experimental economics, bargaining theory and bounded rationality modeling as separate but systematically related topics. His good counsel and support are gratefully acknowledged.

[1] Reinhard Selten has himself worked in this area, see Sauermann and Selten 1962 and Selten 1998.

many experiments in bilateral negotiations by Bartos and Tietz (1983) and Tietz and Werner (1982) leads to a basic structure of aspiration levels of a negotiating person j that may be summed up in the following way:

P_j is the planned goal,

AT_j is the agreement seen as attainable,

AC_j is the lowest acceptable agreement,

T_j is the planned threat to break off negotiation

L_j is the planned break off of negotiation.

The experiments indicate that the preceding five levels should be expected to play a role in practically all real world bargaining processes and the mental processes of a typical bargainer j. The levels are ranked with respect to the preferences of person j such that P_j is the highest level and L_j is the lowest one.[2] If we consider a bargaining problem in monetary space, the intervals between two adjacent aspiration levels of a person form aspiration ranges. In addition we define the lowest aspiration range to be the set that includes alternatives that are ranked below level L, i.e. alternatives where the person would prefer to break off the negotiation instead of accepting the alternative.

The ranking of basic aspiration levels leads to six aspiration ranges. In the case of two persons the intersections of these ranges form an aspiration grid (cf. Figure 1, which is adapted from Tietz and Werner 1982). Figure 1 displays an aspiration structure for persons a and b in the two-dimensional payoff space and shows the feasible set of payoff pairs often used to represent bargaining situations. A certain field in the grid contains all possible agreements that belong to a specified aspiration range of one person and to a specified aspiration range of the other person.

The idea to consider intersections of aspiration ranges is generalizable to bargaining situations with *any structure of the set of alternatives.* In the general case an aspiration range of a person can be defined as a subset of the set of possible agreements X. The set of all aspiration ranges of a person forms a partition of X. In the case of two persons a and b the ranges $A_1, A_2, ...$ form the partition of X for person a and $B_1, B_2, ...$ form a partition for person b. In Figure 2 we have shown a two-person one-dimensional bargaining situation with five aspiration ranges; e.g. the negotiation about the price of a commodity in a bilateral monopoly situation. Here the aspiration sets are not modelled in terms of payoff but define price ranges. Since the bargaining will take place over alternatives in a certain price interval and can only be observed in this dimension, we model the aspiration ranges in the observable variable, too.

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[2] In addition there is an auxiliary aspiration level $B_j \geq P_j$ which represents the maximal payoff or best conceivable result. Tietz and his co-authors asked the participants in the experiments to fill in data for all these aspiration levels. Experimental results by Ahlert 1996 confirmed the relevance of the levels *B, P, AT,* and *AC* in a different design of bargaining experiments.

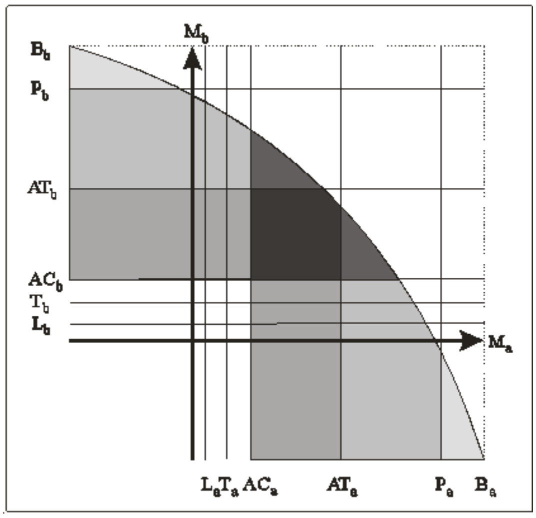

Figure 1: cf. Tietz and Werner 1982

Though most of the experimental evidence comes from bargaining problems where alternatives are characterized in terms of monetary payoffs or one dimensional variables like prices of a commodity, there are also some results about bargaining behavior when alternatives have multidimensional characteristics, e.g. are commodity bundles (cf. e.g. Tietz and Weber 1978). Figure 3 presents an example of a two-person negotiation about two dimensional alternatives. The dimensions could be price and quality as two different characteristics of a commodity. In wage negotiations the dimensions could be wage and working conditions. Whatever the interpretation of the dimensions Figure 3 shows the conflicting interests of both persons and displays possible examples of their different trade offs concerning the two dimensions of the alternatives when they form their aspirations. Of course, the shape of the aspiration ranges depends on how the persons develop their aspirations and can lead to quite different types of aspiration ranges. This two dimensional case can be generalized in the abstract model easily to any number of dimensions of value involved in negotiations.

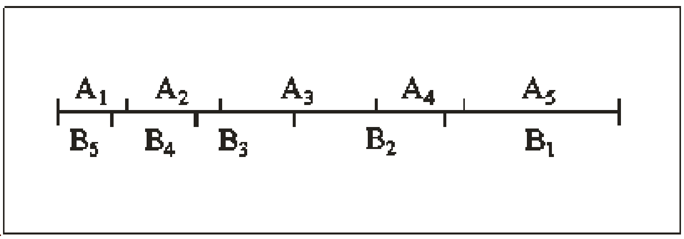

Figure 2: One-dimensional negotiation

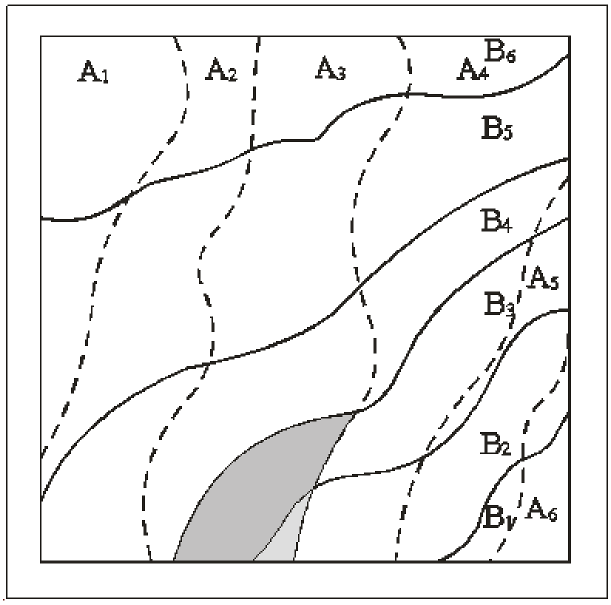

Figure 3: Two-dimensional negotiation

In the following sections of the paper we model a two-person bargaining process and characterize the set of agreements. The behavioral assumptions we formalize in axioms represent features of descriptive theories of bargaining that have been developed from the analysis of experiments. The model of the boundedly rational negotiation process consists of several parts. In section 1 we model the bargaining situation the way it is structured by the negotiating individuals. Then we define a structure of the negotiation process that enables us to present it in a precise notation. In section 2 we develop the axioms that model the boundedly rational bargaining behavior of the individuals. These axioms characterize bounded rationality in bargaining. Section 3 proves the aspiration balancing principle that is well known from descriptive theories as a formal implication of the axiomatic theory. Then we define and characterize the set of possible agreements and also deal with the circumstances for disagreements. The final section 4 contains a general discussion.

1. The Model

A process of bilateral bargaining is modeled for *given aspirations* of the two negotiators. It is left open how the aspirations are formed, how they possibly depend on the framing of the bargaining situation or are influenced by the economic or ethical environment in which the bargaining takes place.

1.1 The Bargaining Situation

Person a and person b are negotiating to find an agreement in a nonempty set X of feasible alternatives. We assume X to be finite, though possibly very large. Since the world is finite this assumption will trivially be fulfilled by sets of alternatives defined in monetary payoffs or by commodity sets.

Definition: *Aspiration Ranges*
A *set of aspiration ranges* of a person j ($j = a$ or $j = b$) is a partition $J_1, ..., J_n$ of X into $n \geq 1$ nonempty subsets of X, i.e.
$J \subseteq X \quad \forall i \in \{1, ..., n\}$ and
$J_i \cap J_k = \emptyset \quad \forall i, k \in \{1, ..., n\}$ with $i \neq k$
and $\cup_{i=1,...,n} J_i = X$.

Experiments indicate that a person will typically have merely five aspiration levels as represented in Figure 1. Together with the auxiliary level B defined by the best outcome and the area below the planned breaking off of negotiations' level L this leads to six aspiration ranges. We model the aspiration ranges such that they are "rank-"ordered according to the preferences of person j: If $i < k$, alternatives in J_i are preferred by individual j to any alternative in J_k. In Figure 1, for instance, the aspiration range with index 1 is defined by the interval between the best level (B) and the planned goal (level P), and the aspiration range with index 6 is everything worse than the planned level to break off negotiations (level L).[3]

Though the subsequent argument holds good for unequal numbers as well we assume for simplicity that both persons have the same number of aspiration ranges. $A_1, ..., A_n$ are the aspiration ranges of person a, and $B_1, ..., B_n$ are the aspiration ranges of person b.

Notation
For any alternative $x \in X$ we consider the index of the aspiration range of person a and the range of person b in which x can be found ($r_a(x)$ or $r_b(x)$, resp.):
$r_a(x) = i \Leftrightarrow x \in A_i$ and $r_b(x) = k \Leftrightarrow x \in B_k$.

Assumption: *Weak Pareto Optimality in Aspirations*
An alternative $x \in X$ with $r_a(x) = j$ and $r_b(x) = k$ will not be proposed if there is an alternative $y \in X$ such that $r_a(y) = j' < j$ and $r_b(y) = k' < k$.

The assumption requires that proposals are not dominated by alternatives such that both persons would be better off. In the case of a one-dimensional negotiation (Figure 2) this assumption is automatically fulfilled. In other cases the set of proposals is restricted to a subset of X (cf. Figure 4 and Figure 5).

Cases of common interests in improving aspiration levels are not considered. Weakly Pareto optimal aspiration range combinations display the conflict of interests between both sides of the negotiating parties.

In the case of a one-dimensional negotiation it is typically the case that the aspiration ranges of the persons are ordered inversely. This is illustrated in

[3] Of course each person has to decide to which of the adjacent ranges the alternative defining the level itself belongs.

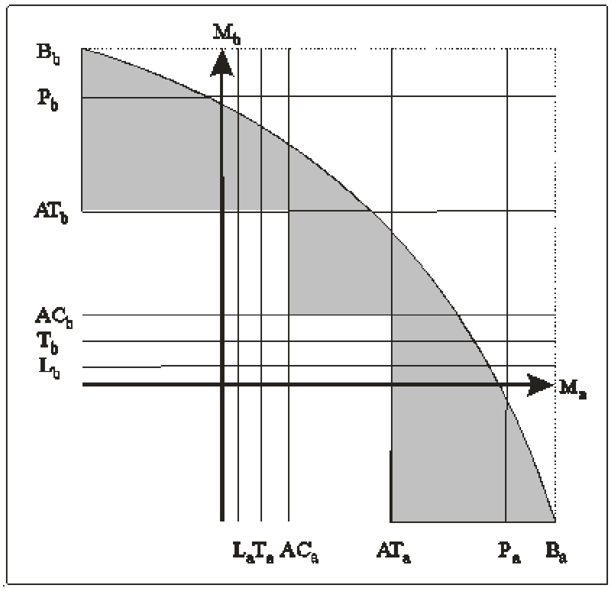

Figure 4: Pareto-optimality in a grid

Figure 2. There e.g. in the aspiration range with index 1 one finds alternatives that belong to the worst range of the opponent. Let us compare two alternatives x and y. If there is an increase of the index of the aspiration range of one person going from x to y we will observe a lower or equal index for the opponent in y as compared to x. In the general case of a multidimensional negotiation this is not necessarily true (c.f. Figure 3). Nevertheless, in Figure 5 we have illustrated a case, where we can always find a path of weakly Pareto optimal proposals in aspirations. For example we find a path that leads from A_1 to A_6 in an increasing order of the indices for person a with step width 1 and simultaneously leads from B_6 to B_1 in weakly monotonic decreasing order of the indices. The analogous observation applies for person b. This motivates how the model proposed here captures the conflict of interest underlying negotiations by its next assumption.

Assumption: *Conflict of Interests*
There is at least one sequence of alternatives $x_1, ..., x_n$ that are weakly Pareto optimal in aspirations, such that $x_j \in A_j$ for all $j = 1, ..., n$ and $r_b(x_j) \geq r_b(x_{j+1})$ for all $j = 1, ..., n-1$. The analogous assumption holds for the aspiration ranges of person b.

1.2 Formal Rules of the Negotiation Process

If negotiation processes are to be theoretically described they must follow at least some rules that define the way proposals are made. We choose the form of

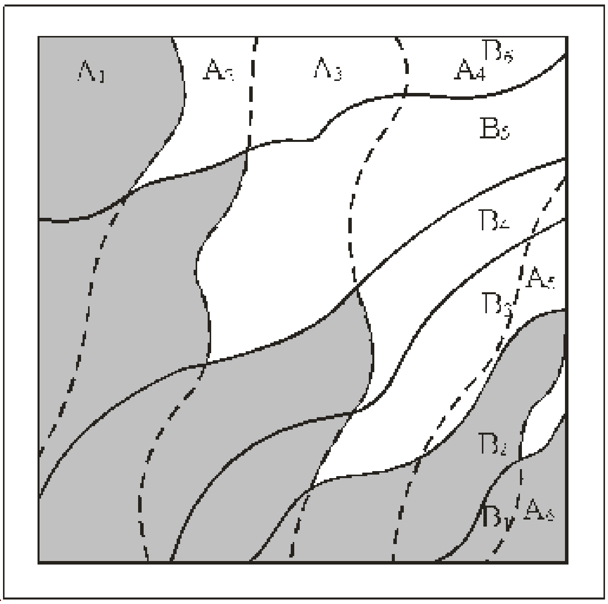

Figure 5: Pareto-optimality in two dimensions

alternating proposals here. This does not necessarily imply a loss of generality. For instance, should a negotiator make several proposals without receiving any proposal of the other person in between then the last proposal in the unilateral sequence may be taken as the one which drives the negotiation forward while neglecting the intermediate ones.

Assumption: *Alternating Proposals*
W.l.o.g. person a starts with the first proposal in X then person b makes a proposal in X and so on until the process stops. Each round of negotiation t with $t = 1, 2, ...$ consists of two proposals, $p_t(a)$ of person a and $p_t(b)$ of person b. The sequence of proposals we consider is $p_1(a), p_1(b), p_2(a), p_2(b), p_3(a), p_3(b), \ldots$where all proposals are elements of X.

For every proposal $p_t\,(a)$ the range of person a is indicated by $r_a\,(p_t\,(a))$ and the range of person b by $r_b\,(p_t\,(a))$. Analogously for a proposal $p_t\,(b)$.

In negotiation experiments we observe that persons repeat a proposal. This often signals that this person is at that stage not willing to give up a certain aspiration range and is waiting for concessions made by the other person. For modeling purposes we need to restrict the number of allowed repetitions to guarantee that negotiation will not end in complete stagnation at some intermediate stage of the process. We assume that there is some (large) number m, such that m is the maximal number of allowed repetitions of the same proposal.

Assumption: *Finite Repetitions*
Let t be any given point in time. Then the cardinality of the sets $\{\, t' \,|\, p_t\,(a) = p_{t'}\,(a) \,\}$ and $\{\, t' \,|\, p_t\,(b) = p_{t'}\,(b) \,\}$ is smaller or equal to m.

This means that no person makes exactly the same proposal of her own more often than m-times, but it is still possible that person a repeats a proposal of person b and vice versa.

The model maps the bargaining process into a four dimensional space. For each point in time we can consider the last proposal of both persons and note the four indices of the aspiration ranges defined by them. The axioms that follow will describe aspects of bounded rationality in bargaining dependent on comparisons of the four indices.

2. Axioms

We start with requirements related to the beginning and the end of the negotiation process.

Axiom: *Start*
Each person chooses the first proposal from her best range, i.e. $r_a(p_1(a)) = 1$ and $r_b(p_1(b)) = 1$.

This axiom is justified by observations in bargaining experiments. It models the fact that negotiators do not give up any of their aspirations without a reason. In the beginning there is no reason yet for a concession. Axiom *Start* does not amount to the assumption, though, that the first proposal will always be the proposer's best alternative. The highest range will contain all alternatives between the best one – as would for instance be envisioned in a classical optimization argument – and the planned goal of an individual seeking to fulfill aspiration levels. The proposer chooses one alternative of them, and in our bounded rationality framework we need not and do not model which one.

The process stops with an agreement, if and only if one of the persons accepts a proposal of the opponent.

Axiom: *Agreement*
The process stops at the minimal t $(= t_0)$ such that

(i) $r_b(p_t(a)) \leq r_b(p_{t-1}(b))$ with the agreement $p_t(a)$ or

(ii) $r_a(p_t(b)) \leq r_a(p_t(a))$ with the agreement $p_t(b)$.

In case (i) b accepts $p_t(a)$. In case (ii) a accepts $p_t(b)$.[4]

Note that in this axiom nothing is said about the index of the aspiration range of the proposer. The condition compares two aspiration ranges of the accepting rather than the proposing negotiator. If the opponent's proposal lies in the same range as the last own proposal of the responding person (i.e. her last demand for herself) then the responder accepts. The aspirations of the accepting person are fulfilled. If the opponent's proposal happens to be in an even better of her ranges (lower index than her own demand) then the responder will of course accept that proposal, too.

[4] b would accept $p_1(a)$, if $r_b(p_1(a)) = 1$, and the process would stop with the first proposal.

The bargaining process ends in disagreement if one of the negotiators would be forced to propose an alternative from her non acceptable aspiration range A_n resp. B_n should the process go on. But by definition of these aspiration ranges each negotiator prefers breaking up the negotiations in disagreement to proposing such an alternative. Therefore in the formal terms of our model, if one negotiator makes a proposal from her range A_n resp. B_n this is not an offer the other negotiator could accept but rather indicates that the negotiation process ends in disagreement.

Axiom: *Disagreement*
The process stops with a disagreement, if and only if person a makes a proposal in A_n or person b makes a proposal in B_n.

The following axioms hold between the start of the negotiation process and its end in agreement or disagreement.

Axiom: *Aspiration Adaptation* (Concessions in own Aspirations)
For both persons $j \in \{a, b\}$ and for all $t = 1, 2, ..., t_0 - 1$:

$r_j\left(p_{t+1}(j)\right) = r_j\left(p_t(j)\right)$ or $r_j\left(p_{t+1}(j)\right) = r_j\left(p_t(j)\right) + 1$ holds.

By the axiom of *Aspiration Adaptation* it is assumed that no person after making some concession will ever revert to making more demanding proposals again. It is also assumed that the maximal concession a person is willing to make comprises one of her own aspiration levels.

The axiom allows for concessions while excluding (inefficient) backward movements during the negotiation. It induces a weak monotonicity on the sequence of aspiration ranges of proposals of the proposing persons themselves. At the same time it leaves room for modeling conditions under which negotiators will give up their actual aspiration range and switch to the range with the next higher index.

In view of the preceding we can state an obvious sufficient condition for the negotiation process to end in finitely many steps.

Lemma: Finiteness of Negotiations
If X is **finite**, then if the assumption ***Finite Repetitions*** and the axioms ***Start***, ***Agreement***, ***Disagreement*** and ***Aspiration Adaptation*** hold the negotiation process stops eventually.

Proof
The proof is done by an argument of finiteness. Consider a sequence of the indices of the own aspiration ranges of proposals of person a. From the axiom *Start* it follows that the sequence starts with index 1. The sequence is weakly monotonic increasing and the maximum step is one index. Since X is finite and repetitions are finite, any aspiration range can be reached after finitely many proposals. If an agreement is reached before person a makes a proposal in her highest range A_n, the process stops with the agreement. If no agreement is reached, the process stops at latest with a disagreement by a proposal of person a in A_n. (Of course person b could be earlier with a proposal in B_n.) In any case the process ends with an agreement or a disagreement. ■

The next axiom excludes a second type of backward movements in the process. In this case we consider the sequence of the aspiration ranges of the opponent that is induced by the sequence of proposals of a negotiator.

Axiom: *Weak Monotonicity of Concessions in Aspiration Ranges of Opponent*
For all $t = 1, 2, ..., t_0 - 1$:
$r_b(p_{t+1}(a)) \leq r_b(p_t(a))$ and $r_a(p_{t+1}(b)) \leq r_a(p_t(b))$.
Concessions that have already been made by some proposer in favor of his opponent are not withdrawn on later stages of the negotiation. Measuring concessions in terms of the aspiration levels of the opponent this axiom models another aspect of bounded rationality in bilateral bargaining. We assume that each proposer will only make a concession, if it is prudent to do so. If a concession was made prudently at some earlier state of the process, there must have been a reason for making it. (We will describe the conditions for those concessions later on.) Withdrawing a concession should lead the opponent to withdrawing a concession, too, since he made it for a reason as well, too. This would lead back to some situation already overcome by a mutually advantageous exchange of concessions. Therefore, if preceding choices have been made prudently, it is imprudent to choose a new proposal with a less generous concession than earlier proposals. We take it that bounded rationality rules out such behavior.

We can now define the admissibility of proposals during the process. This definition will be helpful below.

Definition: *Admissibility*
Given a history of proposals $p_1(a), p_2(a), ..., p_{t-1}(a)$ of person a we call a proposal $p_t(a)$ ***admissible***, if fulfils the assumptions of ***Finite Repetitions*** and ***Weak Pareto Optimality in Aspirations***, and the axioms of ***Aspiration Adaptation*** and ***Weak Monotonicity in Opponents Aspirations*** (analogously for person b).

Admissibility depends only on the former proposals of the proposer.[5] To find admissible proposals (from the weakly Pareto optimal range of combinations) a person need not remember many things. She has to know the aspiration range of her own last proposal and the alternatives from that range that have already been used, since they should not be repeated. She also has to remember the concessions she has already made to the opponent in terms of the opponent's aspiration range, since she should not renege on a concession already made. This is captured in the following lemma.

Lemma: Weak Pareto Optimality of Admissible Proposals
Given a history $p_1(a), ..., p_{t-1}(a)$ of admissible proposals of person a such that $r_a(p_{t-1}(a)) = i$ and $r_b(p_{t-1}(a)) = k$, then $p_t(a)$ is admissible if and only if $p_t(a)$ is weakly Pareto optimal in aspirations and $[p_t(a) \in A_i$ (and does not repeat any proposal $p_1(a), ..., p_{t-1}(a)$ more often than the m-th time) $\vee p_t(a) \in A_{i+1}] \wedge p_t(a) \in \cup_{j=1,...,k} B_j$ holds.

(Analogously for person b)

[5] Of course, in some way or other these proposals will have been influenced by the proposals the opponent has made before his last offer but this is irrelevant here.

Proof

Part 1 ($\Rightarrow$): If $p_t(a)$ is admissible, then *Aspiration Adaptation* implies $p_t(a) \in A_i$ or $p_t(a) \in A_{i+1}$. *Finite Repetitions* implies that $p_t(a) \in A_i$ and it does not repeat any proposal $p_1(a), ..., p_{t-1}(a)$ more often than the m-th time or

$p_t(a) \in A_{i+1}$ and it does not repeat any proposal $p_1(a), ..., p_{t-1}(a)$ more often than the m-th time. This is however equivalent to $p_t(a) \in A_{i+1}$ because *Aspiration Adaptation* implies that $p_1(a), ..., p_{t-1}(a)$ are not in A_{i+1}. *Weak Monotonicity in Opponents Aspirations* implies $r_b(p_t(a)) \leq k$.

Part 2 ($\Leftarrow$):If $p_t(a) \in A_i$ (and it does not repeat any proposal $p_1(a), ..., p_{t-1}(a)$ more often than the m-th time) or $p_t(a) \in A_{i+1}$, person a fulfils *Aspiration Adaptation*.

If $p_1(a), ..., p_{t-1}(a)$ are admissible they are not in A_{i+1}. In both cases *Finite Repetitions* holds. If $p_t(a) \in \cup_{j=1,...,k} B_j$, person a obeys *Weak Monotonicity in Opponent's Aspirations*, since $r_b(p_{t-1}(a)) = k$ ■

Note that *Weak Pareto Optimality in Aspirations* is not implied by *Aspiration Adaptation* and *Weak Monotonicity in Opponent's Aspirations*. In Figures 1 and 3, for instance, one could imagine paths of proposals of stepwise concessions in the required directions including proposals that are not weakly Pareto optimal. Of course these paths could lead to an inefficient agreement.

Now we turn to axioms that model the interaction of the negotiating partners. We will assume that only admissible proposals are made. Both persons compare the size of the concession they have already made in their last proposals in terms of their own aspiration levels. Each person also considers her opponent's aspiration level that is reached by her last proposal and her own aspiration level defined by the opponent's last proposal. In general a proposer will try not to give up the aspiration level she has demanded in her last proposal for herself. This principle is an important part of the descriptive theories of bargaining by Tietz and co-authors (1982; 1983).

Axiom: *Aspiration Securing Principle (for Person* a*)*

If $r_a(p_t(a)) =: i \geq r_b(p_t(b))$

[own concession is not smaller than opponent's concession as measured in her rank terms]

and if $r_a(p_t(b)) > i$

[offer made by b is as things stand not acceptable for a]

and $r_a(p_t(b)) > r_b(p_t(a)) =: k$

[offer person a gets from b is "worse" than offer person a makes to opponent b]

and if there exists $x \in A_i$ such that $p_{t+1}(a) = x$ is admissible,

then person a chooses an admissible $p_{t+1}(a) \in A_i$.

In the situation described in the axiom person a defends (or secures) her aspiration range i. It is a situation where person a has already given up at least as much as person b but is not treated equally well by her opponent. In this case person a will wait until the other person will move by making a better offer. She is not willing to adapt her aspirations to a lower level. Therefore she "insists" by choosing an admissible proposal in the aspiration range of her last proposal, if possible. If no admissible proposal in A_i is available, a concession in person a's own levels is inevitable unless the negotiation ends in disagreement.

The axiom can be formulated analogously for person b. Person b compares her indices of $p_t(a)$ and $p_{t-1}(b)$ and has to choose a proposal $p_t(b)$.

The last axiom constructs situations where persons are not willing to give up their own last aspiration level. In these cases a concession in own aspiration levels is only made, if no admissible proposal in the old level is available. To base a negotiation process only on these forced concessions would mean that the process would as a rule take very long to run its course. In experiments, however, we observe that negotiations move on much faster displaying a type of (bounded) rationality that leads the negotiating partners to make large unforced concessions or "leaps" if the latter are not too "risky".

Axiom: *Prudence of own Concession Making (for Person* a*)*
If there is no agreement in t implied by the axiom *Agreement*, and if $r_a(p_t(a)) < r_b(p_t(b))$, then at time $t+1$ two cases can happen:
(i) If $r_a(p_t(b)) = r_a(p_t(a)) + 1$, then person a agrees to $p_t(b)$.
(ii) If $r_a(p_t(b)) > r_a(p_t(a)) + 1$, then for person a's proposal at time $t+1$ level $r_a(p_t(a)) + 1$ holds (which is possible because of the assumption of *Conflict of Interests*).

The axiom can be formulated analogously for person b.

In the situation described in this axiom person b has already given up more aspiration levels than a. In case (i) person b's offer is "close" in terms of person a's aspirations to a's last demand for herself. Therefore, person a agrees to this offer. In situation (ii) person b's proposal is too far away from person a's demand, so that she will not agree, but negotiator a will make a concession and give up her last aspiration level. First, it is not very risky to do so, since person b is at least one step ahead in her concession, so that person a can be sure not to make a concession that would give up a position too early. Secondly, it is a matter of prudence to do so, since person b has already made a larger concession as measured in her aspiration level terms and person a cannot expect that b will concede more before a has made a concession herself. Person a should not try to force person b to make an even larger concession by insisting on her aspiration level $r_a(p_t(a))$ for period $t+1$ as well. An attempt to use a proposal of the same level for the next period might only lead to a prolonged negotiation process or even to a failure of the negotiation.

The axioms introduced so far do not uniquely fix the proposals the players can make if they follow these rules. Players remain always free to choose an alternative within aspiration ranges. Within a level they can for instance choose alternatives according to their preferences or according to some prominence criterion or other. There are situations where they are even free to choose the next aspiration range. This is especially the case in situations where the levels they both demand for themselves are identical and also the levels they offer to their opponents are the same (though different from their own levels). In such a situation the relative positions of the negotiating players are symmetrical in terms of aspiration levels. Then the next proposer when taking her turn is free to defend her aspiration level or not. Without violating any norms of boundedly rational behavior and quite in line with observational evidence she can decide whether she wants to make a concession in levels of the opponent or not, or whether she wants to make a concession in own aspiration levels or not. Finally, after having made a concession in own levels a person can always defend her new aspiration range as long as it seems necessary to her.

3. The Solutions

From the preceding assumptions and axioms characterizing the behavior of boundedly rational negotiators we derived that their negotiation process, going through a sequence of non-decreasing indices of aspiration ranges of demands, stops after finitely many turns. From the axioms of *Aspiration Adaptation* and *Prudence* it follows that the negotiators starting with their most demanding aspiration range will make progress during the negotiation process by giving up their aspirations stepwise and by making stepwise concessions in terms of the aspiration levels of their opponent. They watch the behavior of the opponent and they respond to the constellation of the four types of aspiration levels defined by the last proposals of both persons. Each person compares her situation to that one of the other person. Among the four indices under consideration, there are two that describe the strength of own demands in terms of own ranges, i.e. what negotiators want for themselves. These ranges have a special importance for the process. From the requirements we impose on their behavior it follows that the negotiators will react in a way such that the aspiration levels they claim for themselves at each state of the process will be identical or will at most differ by one level. This can be captured by the image of a balance that has to be kept in equilibrium. In balance a change on one side requires a similar change on the other side.

Aspiration balancing is a fundamental observation from the experimental data (cf. Bartos and Tietz 1983). In our theory it is derived as a result of axioms characterizing the bounded rationality of concession making.

Lemma: Balancing of Aspiration Levels
As long as the process does not stop, the following holds:
$[r_b(p_t(b)) = r_a(p_t(a))$ or $r_b(p_t(b)) = r_a(p_t(a)) + 1]$ and
$[r_a(p_{t+1}(a)) = r_b(p_t(b))$ or $r_a(p_{t+1}(a)) = r_b(p_t(b)) + 1]$.

Proof
The proof is made by induction over t.
$t = 1:$ Axiom *Start* implies $r_a(p_1(a)) = 1 = r_b(p_1(b))$. *Aspiration Adaptation* restricts $r_a(p_2(a))$ to be equal to $1 = r_b(p_1(b))$ or to $2 = r_b(p_1(b)) + 1$.
$t \to t+1:$
Case 1: $r_b(p_t(b)) = r_a(p_t(a))$ and $r_a(p_{t+1}(a)) = r_b(p_t(b))$. Then (case 1.1) $r_a(p_{t+1}(a)) = r_b(p_{t+1}(b))$ or (case 1.2) $r_a(p_{t+1}(a)) + 1 = r_b(p_{t+1}(b))$ holds. In case (1.1) we have $r_a(p_{t+2}(a)) = r_b(p_{t+1}(b))$ or $r_a(p_{t+2}(a)) = r_b(p_{t+1}(b)) + 1$ by *Aspiration Adaptation*■ In case (1.2) *Prudence of own Concession Making* implies $r_a(p_{t+2}(a)) = r_b(p_{t+1}(b))$ ■
Case 2: $r_b(p_t(b)) = r_a(p_t(a))$ and $r_a(p_{t+1}(a)) = r_b(p_t(b)) + 1$. Here the *Prudence* axiom applied to person b implies $r_b(p_{t+1}(b)) = r_a(p_{t+1}(a))$. *Aspiration*

Adaptation for person a leads to $r_a(p_{t+2}(a)) = r_b(p_{t+1}(b))$ or $r_a(p_{t+2}(a)) = r_b(p_{t+1}(b)) + 1$ ■
Case 3: $r_b(p_t(b)) = r_a(p_t(a)) + 1$ and $r_a(p_{t+1}(a)) = r_b(p_t(b))$. Here we have cases analogously to cases (1.1) and (1.2) ■
Case 4: $r_b(p_t(b)) = r_a(p_t(a)) + 1$ and $r_a(p_{t+1}(a)) = r_b(p_t(b)) + 1$. This case is solved analogously to case 2 ■

The indices of aspiration ranges of any two proposals of different persons that directly follow upon each other in the time sequence cannot differ by more than one. Therefore, considering the complete sequence of proposals of both persons together, we can conclude that we have a weakly monotonic sequence of demanded levels. This means that the situation $r_a(p_t(a)) > r_b(p_t(b))$, for instance, will never occur (only if some player makes a mistake).

With the important property of aspiration balancing in the negotiation process in hand we are now in a position to characterize the set of possible agreements under the behavioral rules defined above.

Definition: ***Set S of Solution Candidates***
We define $s := \min\{\, k \,|(\cup_{i=1,\ldots,k} A_i) \cap (\cup_{j=1,\ldots,k} B_j) \neq \emptyset\}$. s is the smallest index of aspiration ranges of both persons such that there are common alternatives in these ranges or in preferred ones. (In Figures 1 to 3 s is always equal to 3.) The number of aspiration ranges n is an element of $\{\, k \,|\cup_{i=1,\ldots,k} A_i \cap \cup_{j=1,\ldots,k} B_j \neq \emptyset\}$, therefore this set is nonempty. Since it is also finite, the minimum s exists. In view of this we can now define: $S := [A_s \cap \cup_{j=1,\ldots,s} B_j] \cup [B_s \cap \cup_{i=1,\ldots,s} A_i]$.

Due to the definition of s the set S is nonempty. In Figure 1, S is the area that is shaded in the two darkest types of grey.

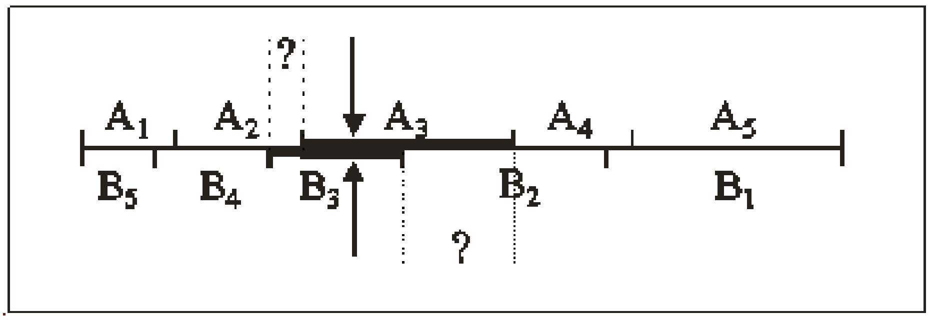

Figure 6: Solutions in a one-dimensional poblem

In Figure 6, s is equal to 3 and S is the area with the bold line including the parts with question marks. In Figure 3, S is the area in the two shades of grey. Note that S is not equal to $A_s \cap B_s$, but may include some more demanding alternatives for each of the persons as long as these alternatives belong to the range with index s of any person. From aspiration balancing one might get the impression that solutions are egalitarian in aspiration ranges. However, solutions in areas like the ones with question marks in Figure 6 are also possible.

The set of agreements is uniquely characterized by the two following theorems as being equal to S. According to the first theorem any agreement solution is an element of S while according to the second any element of S is an agreement emerging as the solution for some conceivable negotiation process.

Theorem: Solutions are in S
Any negotiation process fulfilling the assumptions and axioms above stops at an agreement on some alternative in S.

Proof
Case 1: Let x be a proposal such that $r_a(x) < s$ and $r_b(x) < s$. This is impossible because of the definition of s ■
Case 2: Let x be a proposal such that $r_a(x) = s' < s$ and $r_b(x) = s'' > s$.
Case 2.1: x is proposed by person a.
Aspiration balancing implies $r_b(p_{t-1}(b)) = s'$ or $r_b(p_{t-1}(b)) = s' - 1$. Since s' and $s' - 1$ are both smaller than $s'' - 1$, person b will not accept x, but will make a concession. Therefore, in this case x is no agreement ■
Case 2.2: x is proposed by person b.
Then there must have been a proposal by person b at some time $t' < t$ such that $r_b(p_{t'}(a)) = s$. This implies $r_a(p_{t'}(a)) = s$ or $r_a(p_{t'}(a)) = s - 1$, because of aspiration balancing. Since $p_{t'}(b)$ was not accepted by person a, $r_a(p_{t'}(b)) \geq s$ and therefore $r_a(p_{t'}(b)) > s'$. This is a contradiction to monotonically decreasing aspiration range indices, since $r_a(p_t(b)) = s'$ ■
Case 3: Let x be a proposal such that $r_a(x) > s$ and $r_b(x) > s$. Then x would be dominated by an alternative $y \in S$ such that $r_a(y) \leq s$ and $r_b(y) \leq s$, because S is nonempty. Therefore, x cannot be an agreement. ■

Having shown that the negotiation processes will stop at some alternative in S we prove that each alternative in S can be a solution to some process that fulfils the rules.

Theorem: S consists of Solutions
For any alternative $x \in S$ there is a process that fulfils the assumptions and axioms and stops at x.

Proof
Let $x \in S$ be given.
Case 1: $x \in A_s \cap B_s$. We construct a process by choosing for $t = 1, ..., s-1$ $p_t(a)$ to be an element of A_t and $p_t(b)$ to be an element of B_t such that *Weak Monotonicity in Opponents Aspirations* is fulfilled, but the index of the opponent's range is always larger than t (because of the definition of s). This is possible because of the assumption *Conflict of Interests* in the definition of the bargaining model. In this process there is no agreement possible up to $t = s-1$, the axioms *Start* and *Aspiration Adaptation* are fulfilled, securing of aspirations is not necessary during this process, since *Aspiration Balancing* is fulfilled. At time $t = s$ person a proposes x which is admissible and person b accepts x because of the axiom *Prudence*.
Case 2: $x \in A_s \cap \cup_{j=1,...,s-1} B_j$. Then we construct the process analogously to case 1 up to $t = s - 1$. At time $t = s$ person a proposes x and person b accepts x because of the axiom *Agreement*.

Case 3: $x \in B_s \cap \cup_{j=1,...,s-1} A_j$. In this case we construct the process for both persons similar to case 1 up to time $s-1$ with the change that person b starts with the first proposal. At time $t = s$ person b chooses x and person a agrees because of the axiom *Agreement* ■

If we measure the outcome of the negotiation process in terms of the index of the aspiration range of each person, then it is impossible to find an agreement where both persons' aspiration ranges would have a lower index. This follows from *Weak Pareto Optimality in Aspirations.* It might however be possible to find an agreement where one person keeps her level while the level of the other would be improved. For instance an agreement in Figure 1 in the darkest grey area is weakly Pareto optimal but not strongly Pareto optimal. An agreement in the second darkest area is strongly Pareto optimal. In sum, agreements are weakly Pareto optimal in aspirations but not always strongly Pareto optimal in aspirations.

In the model of the bargaining situation we have assumed that the aspiration ranges A_n resp. B_n contain those alternatives that are not acceptable to person a resp. person b. We assume that the process stops with a disagreement if and only if person a makes a proposal in A_n or person b makes a proposal in B_n. Under the assumptions of the preceding theorems this implies that disagreement emerges if and only if $s = n$. As long as there are alternatives in X that are acceptable to both partners ($s < n$) and if both partners follow the rules modeled in the axioms, then disagreement will never occur. This is as should be among boundedly rational negotiators who within the constraints of their rather coarse methods of aspiration adaptation nevertheless are acting in their best interest. A negotiation process as characterized here is close to what real actors in fact have been observed to do and is therefore in reach of what they can do. So boundedly rational actors may be expected of being able to use the preceding set of axioms as normative standards of what they should do in negotiations. And, if they do, this will be prudent since it will lead to a solution in S and S contains all reasonable and only reasonable solutions of negotiations of boundedly rational negotiators as characterized here.

4. Discussion

This paper axiomatically presents descriptive aspects of bargaining behavior as dependent on aspirations. The main axioms concern the concession making behavior in negotiations. These axioms are formulated as features of boundedly rational behavior of single individuals. From the axioms we could deduce a general property of the bargaining process, i.e. the aspiration balancing principle.

Though this principle can be interpreted as a kind of fairness norm adopted in bargaining we derived it as the result of assumptions characterizing the bounded rationality of individual actors. The aspiration balancing principle is not only close to normative principles like fairness but also to empirical findings about what real human negotiators do in their boundedly rational dealings with each other. The theory developed here axiomatically is always both descriptive and normative. Though the axioms are motivated by observations made in many bargaining experiments and are therefore developed as a formal model of a descriptive theory, they can also be interpreted normatively in the following sense: If both negotiators in a bilateral bargaining situation observe the axioms as rules or standards guiding their choices they will reach a fair and efficient solution in a process that is as long as necessary and as short as possible.

We did not model the formation of aspiration levels. From experiments we know that participants do not bring all of these aspiration levels to the table so to say but rather form them gradually within the negotiation process itself. We know also that given the opportunity participants of bargaining experiments in general try to communicate their aspiration levels once formed. The opportunity to communicate aspirations is not dependent on the ability to communicate in the conventional sense of verbal exchange, though. For instance in so-called "bargaining experiments without communication" participants signal their aspiration levels by repeating a certain proposal several times. This way they "communicate" that they do not want to give up that level. Clearly such signals are subject to falsification. Able negotiators may in fact induce their opponents to endorse false beliefs about themselves. Though they do so at the risk of failing to reach agreement where mutually advantageous agreement with true beliefs may be possible they still may reap some advantage from inducing false beliefs for instance by "bluffing" strategies. Still, whatever the emerging beliefs are, in the end the ensuing negotiation process will have the features described here on the basis of those beliefs. Given these beliefs the process will be in agreement with actual behavior and with the boundedly rational pursuit of the interests of negotiators endorsing such beliefs.

The approach proposed here does not only avoid all full rationality assumptions characteristic of non-cooperative bargaining theory. It also differs from cooperative bargaining models like those in the seminal papers by Nash (1950) and Kalai and Smorodinsky (1975). Though our result has some similarity with an egalitarian solution (equal aspiration ranges plus or minus one level), that result is not derived by any monotonicity consideration comparing representations of different bargaining situations. In our model the egalitarian principle is rather implied by individual prudence and therefore based on a completely different argument.

Our approach is also conceptually different from Reinhard Selten's Negotiation Agreement Area approach, NAA, (cf. Uhlich 1990). The agreement areas of our theory and NAA can have a large set of common alternatives if aspirations are influenced by distributive rules on payoffs that the subjects apply (cf. Klemisch-Ahlert 1996). But NAA is a solution concept for two-person characteristic function games that relies on maximal aspirations and attainable aspiration levels for the players. Boundaries for the agreement area are calculated by using these levels and proportionality or "split the difference" rules along with considerations of prominence.

Whatever the differences between the present and such approaches as NAA may be they are in fundamental agreement that a bounded rather than a traditional full rationality approach should be pursued if we intend to formulate a more realistic bargaining theory. It is of considerable interest for further research to deal in more detail with the relative advantages and disadvantages of alternative approaches of the bounded rationality variety. As in economics in general taking seriously the experimental evidence and the ways real people do and can act is what we should do if real progress in descriptive as well as normative issues rather than cementing some orthodoxy or other is our aspiration.

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